Massively Parallel Machine Deduction on Natural Language (NL) Sentences: A Graphical Approach

Abstract. This study describes a general approach to the assignment of graphs to natural language (NL) sentences on which machines can rapidly approximate the determination of deductive relationships among NL sentences.
A. Our approach is *semantic* in the sense that it uses a notion of interpretation which
(i) assigns set theoretic values to meaning bearing components of sentences as their denotations;
(ii) distinguishes between minimal meaning bearing components (constants) which are assigned the same value by all interpretations, and minimal meaning bearing components (variables) which can be assigned different values by different interpretations,
(iii) assigns set theoretic values to meaning bearing sentence components in such a way that a given sentence is true or false under a given interpretation depending on the pattern of interconnections among those values (assigned it under that interpretation), where the pattern of interconnections among those values is a statement in the meta-language defining the condition under which that sentence is true, referred to as *the truth condition of that sentence relative to that interpretation*, and
(iv) assigns a *denotation to a given sentence* as a set theoretic value wholly determined by the values which that interpretation assigns to its variables and which render that sentence true, such that the set theoretic structure of that denotation wholly determines its deductive connection with other sentences.
B. Our approach is *graphical* in the sense that it assigns graphs to denotations of sentences as pictorial depictions of those denotations with which they are inter-retrievable, and which pictorially depict (as graphs) the deductive relationships that hold among them.
C. Our interest in this paper is on massively parallel machine deduction on natural language (NL) sentences, and is directed toward the use of graphs for this purpose. We proceed by illustrating the major notions of this paper in the familiar areas of elementary algebra and standard form categorical logic, then generalize them for an application to natural language. In this application, we describe a novel type of syntactic structure for natural language character strings capable of supporting semantic structures which are amenable to graphical depictions on which to base massively parallel machine deduction operations on large sets of grammatically varied sentences.

1. Introduction: Basic Concepts Underlying Approach

1.1. *Readings and representations.* An NL character string is regarded in this paper as a sequence of characters from some alphabet, say of English, and blanks, and a *reading* of a given NL character string is regarded as an assignment of a syntactic and semantic structure to that character string which renders that character string meaningful. *By an NL sentence we mean an NL character string which has been assigned a syntactic and semantic structure by a reading relative to which that character string can be true or false in a given context of use.* By a *representation* of an NL sentence we mean a particular syntactic and semantic structure assigned to the character string underlying it.

1.2. *Purpose.* We describe representations for NL sentences relative to given readings as graphical patterns which machines can near instantaneously match to determine their deductive connections. The graphical pattern associated with a given NL sentence is an array of graphical depictions of the denotations of that sentence relative to a range of interpretations under which that sentence is true. Precision of deductive determinations depends on the range of interpretations considered. We use a novel characterization of NL syntax and semantics designed to support our graphical constructions for natural language.

1.3. *NL representations and machine deduction.* It is widely accepted that near instantaneous determination of deductive connections on large numbers of NL sentences would, if possible, need to be based on parallel rather than serial operations. Current logically based approaches to machine deduction operating on NL representations that are rich enough to express the content of a wide range of NL sentences are typically designed to execute in a serial rather than parallel manner[1,2]. Serial execution usually determines deductive entailments of given sentences as those whose representations can be generated by sequential application of deductive rules to representations of the given sentences. Serial execution (on any beyond the simplest types of NL sentences) is slow because deductive rule applications are interdependent and their order is not determinate, so that heuristics of some sort need to be applied to ferret out unproductive strategies which tend to proliferate the generation of meaningless intervening forms. The larger the number of the given sentences, the greater the problem of proliferation of meaningless intervening forms. As the number of NL sentences increases, ever more complex heuristic devices are needed to restrict the application of deductive rules to just those sentences which appear to be most "relevant" to particular cases, a factor which introduces a further level of indeterminacy. Problems of this kind attending serial execution have rendered syntactically based machine deduction techniques unsuited for real-time execution on large numbers of sentences.

Footnote 1. The most developed approaches using serial inference are those using some form of resolution logic as initially formulated by A. J, Robinson [11] and subsequently substantively developed by others [1].

Footnore 2. The most developed approaches using parallel inference are those using some form of activation logic as developed by Shruti, and his followers. Their application to simple sample sentences [12], [13] , while effective on those sentences, do not appear to be readily generalizable to a sentences of varied grammatical constructions.

1.4. *Requirements for massively parallel NL deduction.* Whatever type of representation used for NL sentences, it should be able to support a deductive mechanism such that all subtasks involved in determining whether a given sentence is deductively entailed by a given set of sentences have the following features: (1) they *are independent of each other in their application and can be executed simultaneously*, and (2) they *can be executed "globally," that is, executed on the set of sentences as a whole" and without regard to their relevance or the size of the set on which they are executed."* (1) means that all subtasks are executed in *parallel, while* (2) means that their execution is *massive.* We claim that semantic representations of NL sentences are better suited for designing these features into their operation than are syntactic representations.

1.5. *A graphical requirement for massively parallel NL deduction. G*raphical depictions of sentence structure must also meet the following additional requirement for massively parallel NL deduction: (3) *The patterns formed by graphical depictions of the semantic structures of given NL sentences need to approximate the deductive connections that hold* *among those sentences, and to do so in a manner that a machine can identify near-instantaneously*. (3) means that such graphical patterns must exhibit deductive connections in a manner that a machine can near instantaneously identify.

1.6. *Syntactic and semantic representations of NL sentences.* A *syntactic representation of an NL sentence* is a linguistic structure which identifies the meaning bearing parts of the sentence, and recursively composes them to form the sentence as a whole, and a *semantic representation of an NL sentence* is a set theoretic[3] structure recursively composed of elements of the domain of discourse and sets composed of them which are assigned to its meaning bearing parts as their "meanings."
Any syntactic or semantic representation of NL sentences can be graphically depicted to some extent. The question is whether that graphical depiction can be used as a basis for identifying deductive entailments and, more particularly, the extent to which they can satisfy requirements (1), (2), and (3). We claim that the semantic representations of NL sentences and the graphical depictions based on them[4] are defined in this paper in such a way as to satisfy these requirements.

Footnote 3. By a set theoretic structure I mean an element in the domain of discourse or a set constructed from such elements by application of the classical rules of set formation.

Footnote 4. See Sections 5.1. – 5.5, below.

2. Approach

2.1. *Two examples: elementary algebra and standard form categorical logic.* We briefly consider two simple examples familiar from the classroom whose semantic structure satisfies the structural requirements (1) and (2), and whose graphical structure satisfies the graphical requirement (3). The first example describes how *subtasks involved in determining deductive connections among all equations and inequalities in a given set are independent of each other in their application and executable simultaneously*, and are *executable "globally," (hence satisfying both structural requirements),* and that they are such that the patterns formed by graphical depictions of the semantic structures of given equations and inequalities enable near instantaneous machine approximation[5] of the deductive connections that hold among them, (hence satisfying the graphical requirement as well). The second example describes how *subtasks involved in determining deductive connections among standard form categorical sentences in a given set are independent of each other in their application and executable simultaneously*, are *executed "globally," (hence satisfying both structural requirements),* and that they are such that the patterns formed by graphical depictions of the semantic structures of given standard form categorical sentences enable near instantaneous machine determination[6] of the deductive connections that hold among them are sufficient to determine (rather than only approximate) the deductive connections that hold among them (hence satisfying the graphical requirement as well)..

Footnote 5. The degree of approximation – in the case of elementary algebra) depends, as would be expected, on the pixel size used.

Footnote 6. In the case of standard form categorical sentences the graphical determination of deductive connections is not approximate.

2.2. *Natural language.* We extend the ideas implicit in these two classroom examples to natural language (NL) by describing certain semantically based graphs for NL sentences which, in analogy to the above two examples, satisfy requirements (1), (2), and (3). That is, we describe how *subtasks involved in determining deductive connections among NL sentences in a given set are: (1) independent of each other in their application and executable simultaneously*, (2) *executed "globally," that is, executed on all NL sentences in the set* and (3) are such that *the patterns formed by graphical depictions of the semantic structures of given NL sentences are sufficient to enable a machine to at least approximate*[7] *the deductive connections that hold among them, and to do so near-instantaneously.* We will refer to the graphs for NL sentences as *generalized node-and-arrow (GNA) graphs*.[8]

Footnote 7. The degree of approximation – in the case of NL sentences depends on constraints placed on the range of interpretations used, hence on the range of the graphical structures which depict them.

Footnote 8. "Node-and-arrow" graphs of different types have been used in the logical and computer science literature to depict restricted logical relationship among simple NL sentences, they have characteristically been applied to the simplest type of NL sentences, and without providing a theoretical account of their underlying rationale which could be extended to more complex cases. GNA graphs described here are designed to do this, that is, to apply across varied NL sentences of arbitrary grammatical types.

2.3. *Generic notion of sentence.* In order to make our account equally applicable to these three cases – equations and inequalities of elementary algebra, standard form categorical sentences, and sentences of natural languages like English – we use the term "sentence" to apply to any expression of a language which, relative to a particular way of assigning set theoretic meanings to its meaning bearing parts, can be assigned a truth value. In this sense, the equations and inequalities of elementary algebra qualify as sentences inasmuch as the variables they contain can be assigned real numbers as their set theoretic[2] meanings and, recursively, so can algebraic expressions built out of them using the customary algebraic constants.

2.4. *Local and global denotations and graphs*. We distinguish the denotation of a sentence relative to a given interpretation as its *local denotation relative to that interpretation, and* distinguish the set of all local denotations of that sentence relative to all (permissible) interpretations as the *global denotation* of that sentence. We distinguish the graph assigned to a local denotation of a sentence as a *local graph* of that sentence, and distinguish the graph assigned to the set of local denotations of that sentence as the *global graph* of that sentence, which is a graphical array of its local graphs. For example, in the special case of elementary algebra of the plane, a *local denotation* of an equation or inequality is an ordered pair of real numbers usually referred to as its "solution," its *global denotation* is the set of all its solutions usually referred to as its "solution set," its *local graph* is a point on the plane, and its global graph is the array of all its local graphs on the plane.

2.5. *Semantic and Graphical Deductive Paradigms.* We describe three *semantic* deductive paradigms, and three graphical paradigms based on them. One is *strong and positive*, another is *weak and positive*, and a third is *negative*. Let P1, ..., Pn, C, and not-C be NL sentences.

2.5.1. The *strong positive semantic paradigm* states that C is deducible from P1, ..., Pn if and only if the intersection of the global denotations of the (premise) sentences, P1, ..., Pn, is a subset of the global denotation of the (conclusion) sentence C.

2.5.2. The *strong positive graphical paradigm* states that C is deducible from P1, ..., Pn if and only if the graphical intersection of the global graphs of P1, ..., Pn is a subgraph of the global graph of C.

2.5.3. The *weak positive semantic paradigm* states that C is deducible from P1, ..., Pn if every consistent selection of local denotations of P1, ..., Pn, one local denotation from each of P1, ..., Pn, sententially implies some local denotation of C.[9]

2.5.4. The *weak positive graphical paradigm* states that C is deducible from P1, ..., Pn if every consistent join of the local graphs of P1, ..., Pn, one local graph from each of P1, ..., Pn, graphically contains some local graph of C. [10]

2.5.5., The *negative semantic paradigm* states that C is deducible from P1, ..., Pn if and only if every local denotation of not-C is inconsistent with every selection of local denotations, one local denotation from each of P1, ..., Pn. [9]

2.5.6. The *negative graphical paradigm* states that C is deducible from P1, ..., Pn if and only if every local graph of not-C is *graphically incompatible* with every join of the graphs of P1, ..., Pn, one local graph from each of P1, ..., Pn.. [11]

Footnote 9. To render these two semantic parahgms precise would require the description of a conversion rule which converts denotations into a suitable sentential form. Since our dominant interest in this paper is with the use of graphs in machine deduction, we do not describe a conversion rule for them, though such a description would be fairly straightforward.

Footnote 10. Note also that the strong positive graphical paradigm does not imply the weak positive graphical paradigm, Rather, if both the strong and the weak positive graphical paradigms hold, then the graph of (the conclusion) C *is identical with the graphical join* of the graphs of (the premises) P1, ... Pn.

2.6.The key notions in these graphical paradigms are the notions of *graphical join, graphical incompatibility, graphical subgraph, and graphical inclusion which we can characterize as follows:* (i) The *graphical join between two or more between consistent local graphs* is a configuration formed by linking them with dotted lines or dashed lines, and graphically depicts the conjunction of the local denotations which those local graphs depict. (ii) Two or more local graphs are *graphically compatible or* consistent if the truth conditions of the local denotations they depict are consistent. (iii) One local graph is a subgraph of another if it is contained wholly within it. (iv) One local graph is included in another if it is a subgraph of it.[11, 12]

Footnote 11. These notions are not precise. Their precise statement requires a more thorough account of the structure of the graphs used, which goes beyond the scope of the present paper. Their meaning, however, can be intuitively understood in our examples to follow.

Footnote 12.. In order to have the strong semantic and graphical paradigms apply to categorical syllogisms and coordinate algebra, we can use the following notion of cylindrical projection: Let V be a set of variables of a language L, let F be a set of interpretations f on the elements of V on a set of variables $V = \{v_1, \ldots, v_n\}$, let v_i be an element of V, and let f be an element of F: Then the cylindrification Cyl(V, v_i , f, F) (read as the cylindrification of V relative to f, v_i ,and F) is the set $\{f(a_1), \ldots, f(a_{i-1}), f^*(a_i), f(a_{i+1}), \ldots f(a_n)\}$ where f* is an element of F.and a_i is the set {we represent the content of each categorical sentence in a manner that makes also reference to the term not contained in it. This sort of procedure in well known in algebra, in particular, in representing an equation or inequality having two variables in 3-space, whereby its solution set includes reference also to all possible values of the missing variable. For example, the equation $y = x^2$, whose graph is a parabola on the x-y plane, can be equivalently written as $y = x^2$ & $z = z$, the equation in this form having as its graph a cylindrical surface parallel to the z-axis and intersecting the parabola $y = x^2$on the x-y plane.
This process, referred to in the literature as "cylindrification," is applicable to a variety of suitably formalized languages. We apply it to the language of standard form categorical sentences by replacing, say, the categorical sentence "All X are Y," by the equivalent (though non-categorical) sentence "All X are Y and Z is Z," and graphically by replacing the two circle Venn-diagram of the sentence. "All X are Y" by the usual circle Venn-diagram.

2.7. *Application of deductive graphical paradigms to elementary algebra:* The notions of join, graphical inclusion and graphical incompatibility have a definite meaning in the usual graphing structures of elementary algebra familiar from the classroom, where *the graphical join* of graphs *is their graphical intersection*, *graphical inclusion is the containment of one graph wholly within another and graphical incompatibility is the disjointedness of one graph from another*.

2.8. *Application of deductive graphical paradigms to standard form categorical logic.* In their usual classroom treatment, Venn diagram graphs do not lend themselves readily to defining graphical join as graphical intersection, defining graphical inclusion as graphical containment, or defining graphical incompatibility as graphical disjointedness. By using an alternative (but equivalent) version of Venn diagrams, we can define graphical join, graphical inclusion, and graphical disjointedness in a manner which parallels their definitions in elementary algebra, and which is also applicable to arbitrary finite numbers of standard form categorical sentences (i.e., not just three, as used in customary evaluations of valid syllogisms).[13]

Footnote 13. See Section 4, below.

2.9. *Application of deductive graphical paradigms to natural language:* Graphs of NL sentences as presented in this paper are defined in such a way that notions of graphical join, graphical inclusion, and graphical incompatibility can be defined for them that would enable both the positive and negative deductive graphical paradigms.

2.10. *Towards a common vocabulary.* To apply these positive and negative deductive paradigms to the sentences of elementary algebra, categorical logic, and natural language, we introduce a vocabulary applicable to all three. Toward this end, we have already remarked (in Section 2.3, above) on the general notion of "sentence" we will be using. We remark here on other notions used generally, that is, independently of particular applications. These notions are:.
2.10.1. *Interpretations.* An *interpretation* of an expression contains two components. One is a *syntactic component* which recursively identifies the meaning bearing parts of the expression and the mode of their composition to form the expression as a whole. We refer

to the output of the syntactic component of an expression as its *syntactic structure*. The other component is a *semantic component* which recursively assigns a set theoretic structure to every meaning bearing part as its meaning. We refer to the set theoretic structure assigned to the expression as a whole as its *semantic structure* or *denotation*. We do not describe the syntactic and semantic structures of sentences of elementary algebra or of categorical logic in a precise way, but rely instead on the informal way those structures are usually presented in the classroom. For natural language we need to treat these notions of structure more precisely

2.10.2. An *interpretation family for a language* is a set of interpretations with the same syntactic component. Interpretations in a given family are said to be *permissible*. (See Section 10.3. below for a more detailed description of the notion of permissible interpretation as is applies to a particular natural language sentence.)

2.10.3. *Constants and variables*. An interpretation family divides the expressions of the language into two types: those which are *syntactically simple* and those which are *syntactically compound*, (i.e., composed of simpler expressions); and divides the meaning bearing expressions of the language also into two kinds: those which are assigned the same meaning by every interpretation in the family and are called *constant expressions*, and those which can be assigned different meanings by different interpretations in the family, and are called *variable expressions. Syntactically simple constant expressions are called constants and syntactically simple variable expressions are called variables*.

2.10.4. *Interpretations are distinguished only by their assignments to variables.* Since the expression constants in a sentence are assigned the same meaning by all interpretations, interpretations are distinguished only by what they assign to variables, so that, in order to distinguish among different interpretations of an expression, it suffices to distinguish among the meanings they assign to the variables in that expression.

2.10.5. A *local denotation of an expression* is its denotation relative to a given interpretation is the semantic structure assigned to that expression by that interpretation, and the *global denotation of an expression relative to a given family of interpretations* is the set of its local denotations relative to all interpretations.

2.10.6. *Graphs.* A *local graph of an expression* is a pictorial depiction of its local denotation relative to some interpretation, and its *global graph* is a linked array of its local graphs.

3. Graph Based Machine Deduction on Equations and Inequalities of Elementary Algebra

3.1. *Sentences of elementary algebra of the plane.* A *sentence of elementary algebra of the plane in the variables "x" and "y"*, (i.e., an equation or inequality familiar from the classroom), is composed of the variables "x" and "y", and constants which are, variously, names of real numbers ("2," "e," etc.), function symbols ("+", "-", · etc.), and relation symbols ("=", "<", ">," etc. and their negations).

3.2.. *Interpretations, truth, and graphs of sentences of elementary algebra of the plane in the variables "x" and "y".* An *interpretation* for elementary algebra of the plane is a function f which (i) assigns real numbers f(“x”) and f(“y”), respectively, to the variables “x” and “y”, (ii) assigns real numbers and real valued functions to constants, (iii) assigns relations among real numbers to relation symbols, and (iv) assigns the ordered pair <f(“x”), f(“y”)> to the ordered pair <”x”, “y”>. An equation or inequality E(x,y) is *true under an interpretation f* if and only if the result of replacing the variables “x,” and “y,” respectively by (names of) the real numbers f(“x”) and f(“y”), renders E(x,y) as a true sentence of elementary algebra. The *graph* of the ordered pair <f(“x”), f(“y”)> of real numbers is a point G(<f(“x”), f(“y”)>) on the (Cartesian rectangular) plane such that f(“x”) is the signed distance of G(<f(“x”), f(“y”)>) from the y axis of the plane, and f(“y”) is the signed distance of G(<f(“x”), f(“y”)>) from the x- axis of the plane.

3.3. *Local and global denotations and local and global graphs of equations and inequalities.* A *local denotation of an equation or inequality of elementary algebra* in the variables “x” and “y” *relative to a given interpretation f* is the pair <f(”x”), f(“y”)> of real numbers if that equation or inequality is true when the names of f(”x”), f(“y”) replace the variables “x” and “y” in that equation or inequality (usually referred to as a *solution*), and is the empty set otherwise. The *global denotation of that equation or inequality* is the set of all its local denotations (usually referred to as its *solution set*). A *local graph* of an equation or inequality is a *point* on the plane corresponding to (i.e., which graphically depicts) its local denotation relative to a given interpretation. The *global graph* of an equation or inequality is the array of points on the plane corresponding to pairs of real numbers in its global denotation, and is what is referred to as its “graph” in the classroom.[14, 15]

Footnote 14. The array of points on the plane can be considered a “linked” array, the linkage constituted by the circumstance that the points in the array are on the same plane, each having an orientation relative to the coordinate axes of the plane and, derivatively, to the other points in the array.

Footnote 15. In a formalization of these concepts we would need recourse to a metalanguage in which to define these concepts, which we do not treat here. For example, the notion of equations and inequalities becoming true under replacement of their variables by names of real numbers would be treated under the general operation of translating the object language into a suitable metalanguage for elementary arithmetic.

3.4. *Positive graphical paradigm for elementary algebra*: a given equation or inequality is a deductive consequence of a system (i.e., a set) of equations and inequalities if and only if the graphical intersection of the global graphs of the equations and inequalities in the system is graphically included in the global graph of the given sentence.

3.5. *Negative graphical paradigm for elementary algebra*: a given equation or inequality is a deductive consequence of a system (i.e., a set) of equations and inequalities if and only if the graphical intersection of the global graphs of the equations and inequalities in the system is graphically disjoint from the global graph of the negation of the given sentence.

3.6. *Graph based machine deduction for elementary algebra.* That a machine could simultaneously execute operations on all local graphs in a given global graph to make rapid determinations is familiar from classroom algebra, particularly where graphing calculators are used to rapidly verify (approximately) that a given equation or inequality is a deductive consequence of a system of equations and inequalities by determining whether the graph of the system (the graphical intersection of the sentences it contains) is graphically included in the graph of the given equation or inequality, a determination made by matching the pixels occurring in the graph of the system against the pixels occurring in the graph of the given equation or inequality. The degree of precision of such a determination would depend, of course, on the pixel size used. The advantages of using graph based machine deduction increase as the size of the system increases and are even more marked in making deductive determinations in 3-dimension elementary algebra.

4. Graph Based Machine Deduction on Standard Form Categorical Sentences

4.1. A *standard form categorical sentence* (i.e., as familiar from the classroom) is composed of variables and constants. Constants of categorical logic include the quantifiers "All," "Some," and "No," and the relation constants "Are" and "Are Not." Variables of categorical logic (as we shall characterize them) will be capital English letters with or without subscripts. Variables of categorical logic are capital English letters with or without subscripts. We define a *standard form categorical sentence in the variables V1,V2* as a sentence of one of the following four forms: (i) Universal affirmative (All V1 are V2), (ii) Universal negative (No V1 are V2), (iii) Particular affirmative (Some V1 are V2), and (iv) Particular negative (Some V1 are not V2). We then define a *standard form categorical sentence in two of three variables V1 ,V2, V3* , as any standard form categorical sentence in any two of these three variables.

4.2. *Interpretations and truth for the language of standard form categorical logic in the three variables "V1," V2,"and "VZ".* An *interpretation for a standard form categorical sentence in two of three variables V1, V2, V3*, say, VI, VJ, is a function f which: (i) assigns a set f("V1"), f("V2"), f("V3"), to the variables V1, V2, V3, respectively, which are such that each of the sets f("V1"), f("V2"), f("V3"), forms a non-empty intersection with exactly two of the other two sets, where all three sets intersect non-emptily, and where each set of the three contains elements which are not in the other two, and which (ii) assigns an ordered triple <(f("V1"), f("V2"), f("V3")> of non-empty sets to the variables V1, V2, V3. A categorical sentence in two of three variables "V1, "V2," "V3," say the variables VJ, VK, is *true under an interpretation f* if the relationship which that sentence states as holding between the sets f(VJ) and VK is true.

4.3. *Need for an alternative systems of graphs for standard form categorical logic.* We describe an alternative system of graphs for standard form categorical sentences which is equivalent to the usual Venn diagram graphs in their application to syllogisms but which has the advantage of enabling straight-forward definitions of *graphical inclusion, graphical incompatibility, and graphical combination* and, as a consequence, enabling definitions of both the strong form of the positive deductive graphical paradigm and the negative deductive graphical paradigm to apply in a manner wholly analogous to the way these paradigms apply to the usual graphs of elementary algebra, as stated above in Sections 2.1 and 2.2. This alternative system has the additional advantage of being readily generalizable to arbitrary large finite sets of standard form categorical sentences. As in the case of elementary algebra of the plane, each of the two strong deductive paradigms can be used to simultaneously execute operations on graphs of sets of standard form categorical sentences to make rapid determinations that certain of those sentences are deductive consequences of others. In the following sections, we define the alternative versions of graphs of categorical sentences which enable this.

4.4. *Usual Venn diagram graphs for determining validity of syllogisms.* In application to the determination of the validity of a given syllogism consisting of three standard form categorical sentences (i.e., sentences of the form, "All men are mortal," Some men are mortal," etc.), containing three terms in all[3], each of three overlapping circles graphically representing exactly one these three terms.

4.5. *Alternative version of usual Venn diagram graphs.* Our discussion centers about the proposed alternative version of Venn diagram graphs designed to better accommodate the deductive paradigms by enabling the definitions of graphical inclusion, graphical intersection, and graphical disjointedness as a special case revolving about their use for determining validity of categorical syllogisms as a running example. A virtue of the proposed alternative version (besides enabling the statement of the paradigms) is that it readily applies to any finite number of standard form categorical sentences (rather than just three, as occur in a syllogism). *Assuming that we already have these notions at hand for categorical logic, and that P1, ..., Pn, C are standard form categorical sentences, the application of these two paradigms to categorical logic can be stated as follows:*
4.5.1. *Positive graphical deductive paradigm for categorical logic*: a given standard form categorical sentence is a deductive consequence of a set of categorical sentences if and only if the graphical intersection of the graphs of the categorical sentences in the set is graphically included in the graph of the given categorical sentence.
4.5.2. *Negative graphical deductive paradigm for categorical logic*: a given standard form categorical sentence is a deductive consequence of a set of standard form categorical sentences if and only if the graphical intersection of the graphs of the categorical sentences in the set is graphically disjoint from the graph of the negation of the given standard form categorical sentence.

4.6. *Definition of the graphing function G.* Let f be an interpretation which assigns a set to each of the variables "V1, "V2," "V3," and let G_f be a function that assigns a graph to each of the sets f("V1"), f("V2"), and f("V3"). The *graph G(*<(f("V1"), f("V2"), f("V3")>) of the ordered triple of sets f("V1"), f("V2"), f("V3") is the graphical intersection of the graphs G_f ("V1"), G_f ("V2"), and G_f ("V3")of the those sets. The graphical structure *G(*<(f("V1"), f("V2"), f("V3")>) is usually depicted as a configuration of three overlapping circles graphically depicting the sets f("V1"), f("V2"), f("V3"), such that each circle overlaps part of each of the other two graphically depicting their intersection, and such that the common overlapping part of the three circles graphically depicts the common intersection of the three sets. (See Figure 1.) Thus, the truth of a given a categorical sentence S in two of three variables “V1, “V2,” “V3,” say the variables VJ, VK, is graphically depicted within the graph (G/f)(<(f(V1), f(V2), f(V3)> by the graphical relation holding between the circles depicting the sets f(VJ) and f(VK).

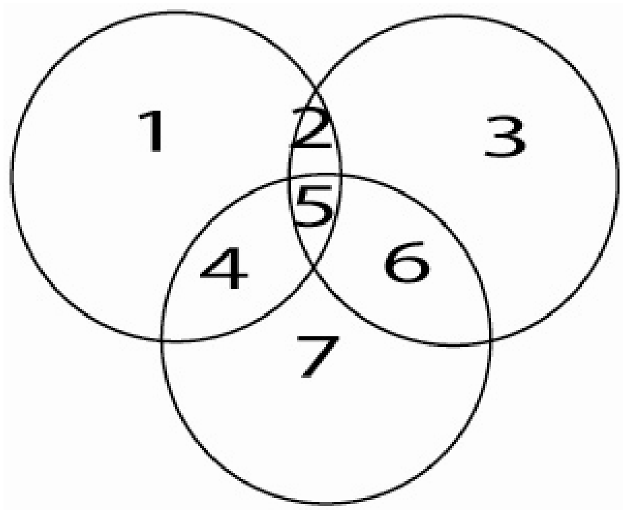


Figure 1

4.7.. *Difficulty with defining G(*<(f(“V1”), f(“V2”), f(“V3”)>). The difficulty with defining the graph of an ordered triple of sets (f(“V1”), f(“V2”), f(“V3”)> as a configuration of three overlapping circles is that it does not exhibit graphical intersection, inclusion, and disjointedness relations among the overlapping circles in a manner required for near-instantaneous execution of the positive and negative graphical paradigms. For this purpose we need to define the graph of an ordered triple of sets f(“V1”), f(“V2”), and f(“V3”)> in a manner which enables the definitions of graphical intersection and inclusion that occur in the statement of the positive and negative graphical paradigms as applied to categorical sentences.

4.8. We will shortly (see 4.9. below) consider an alternative definition of the graphing function (G/f) which does not have these difficulties. Consider the following Boolean combinations of the sets f(X), f(Y), and f(Z):

$r1_f = f(X) - (f(Y) \cup f(Z))$;

$r2_f = (f(X) \wedge (f(Y)) - f(Z)$;

$r3_f = (f(Y) - (f(X) \cup f(Z))$;

$r4_f = (f(X) \wedge (f(Z)) - f(Y);$

$r5_f = (f(X) \wedge f(Y)) \wedge f(Z));$

$r6_f = (f(Y) \wedge f(Z)) - f(X));$

$r7_f = f(Z) – (f(X) U f(Y)).$

$r1_f, ..., r7_f$ are, of course the minimal intersections of the three sets f(X), f(Y), and f(Z), each of which forms a non-empty intersection with two others, where all three sets intersect non-emptily. and where each set of the three contains elements not in the other two.

4.9. *The interpretation function f^.* We define an interpretation for a standard form categorical sentence S in two of three variables V1, V2, V3, as a function f^ which assigns the same sets to the variables V1, V2, V3, as f does, but which assigns the ordered septuple $<r1_f, ... , r7_f>$ of sets to the ordered triple <V1, V2, V3> (instead of assigning it the ordered triple <(f(V1), fV2), f(V3)>).

4.10. The *local denotation* of S relative to the interpretation f^ is the septuple $<r1_f, ... , r7_f>$, and the global denotation of S is the set of all such septuples as f ranges ovcr all permissible interpretations.

4.11. *The graphing function (G/f^).* While intersection, inclusion, and disjointedness relations could be defined on the septuples of the sets $<r1_f, ..., r7_f>$, the graphical relations of inclusion, intersection, and disjointedness are more readily determined by machine if we equivalently depict them as septuples of 0's and 1's, as follows: Let h_f be a function on $r1_f, ..., r7_f$, such that, for $1 \leq i \leq 7$, h_f assigns 0 to ri_f if ri_f is empty, and assigns 1 to ri_f if ri_f is non-empty. We then define *a graphing function (G/f^) as a function which assigns a septuple* $<h_f(r1_f), ..., h(r7_f)>$ *to the ordered triple <(f(V1), f(V2), f(V3)> under the interpretation f*, and refer to it as *the septuple determined by f.* Thus (G/f^)(*<(f(V1), f(V2), f(V3)>)* is a set of septuples of 0's and 1's such that the ith term of the septuple is 0 or 1 according as ri_f is empty or non-empty.

4.12. *Parallels with notions of elementary algebra.* Recalling the notions of point, solution, and solution set of elementary algebra, we regard local denotations as analogous to "solutions," global denotations as analogous to "solution sets," local graphs as analogous to "points," and global graphs as analogous to graphs of solution sets.

4.13. Recall that a local graph of an expression is a pictorial depiction of its local denotation relative to some interpretation, and that its global graph is an array of its local graphs. Accordingly, we define the *local graph of a standard form categorical sentence S* relative to the graphing function G(f^) as a pictorial depiction of its local denotation relative to f^, namely, a pictorial depiction of the septuple <$r1_f$, ... , $r7_f$>, which we defined above as septuple of 0's and 1's such that the ith term of the septuple is 0 if ri_f is empty, and is 1 if ri_f is non-empty, and the *global graph of S* as the set of local graphs of S as f ranges ovcr all permissible interpretations. Thus the local denotation of S relative to f is a septuple of sets while the local graph corresponding to this local denotation is a septuple of 0's and 1's.[16]

Footnote 16. We can generalize this construction to finite sets of categorical sentences of arbitrary size. Let K be a finite set of standard form categorical sentences, and let <V1, V2, ..., Vn > be an ordering of the n variables that occur in sentences of K. An interpretation f on the language of categorical logic can then be defined as a function which assigns, to the sequence of variables <V1, V2, ..., Vn >, the sequence f(V1), f(V2), ..., f(Vn)> which is graphically depicted as an n-Venn Diagram (in the sense of Grunbaum [5]) whose minimal regions r_1, r_2, ..., r_{2n-1} are the 2n – 1 disjoint intersections among the sets f(V1), f(V2), ..., f(Vn).

4.14. *Relationship of f^ to f. W*e note that, while f(X) = f^(X), f(Y) = f^(Y), and f(Z) = f^(Z), we have that f(<X, Y, Z>) is a triple of sets f(X), f(Y), f(Z), such that each forms a non-empty intersection with exactly two other sets of the three, where all three sets intersect non-emptily, and where each set of the three contains elements not in the other two, whereas f^(X, Y, Z) is a septuplc <$r1_f$, ..., $r7_f$ > of the sets defined on f(X), f(Y), f(Z), as above). We note that f(X) = $r1_f$ U $r2_f$ U $r4_f$ U $r5_f$, f(Y) = $r2_f$ U $r3_f$ U $r5_f$ U $r6_f$, and f(Z) = $r4_f$ U $r5_f$ U $r6_f$ U $r7_f$.

4.15. *Truth of a standard form categorical sentence.* Let S be a standard form categorical sentence in the variables VJ, VK, and let f be an interpretation for S. Then:
(1) if S has the form "All VJ are VK," then S is true under f if and only if the set f(VJ) is included in the set f(VK);
(2) if S has the form "Some VJ are VK," then S is true under f if and only if the intersection of the sets f(VJ) and f(VK) is non-empty;
(3) if S has the form "No P are C," then S is true under f if and only if the intersection of f(VJ) and f(VK) is empty;
(4) if S has the form "Some VJ are not VK," then S is true under f if and only if the intersection of f(J) with the complement of f(K) is non-empty[17]

Footnote 17. We could also formalize the language of standard form categorical logic in two variables X and Y in a manner that assimilates the quantifiers "all," "some," and "no" within the relations "are" and "are not," as follows: R1, R2, R3, and R4 are relational constants, specifically, binary relations such that for all sets X and Y of elements in the universe of discourse:
<X,Y> ε f(R1) if and only if X is included in Y;
<X,Y> ε f(R2) if and only if the intersection of X and Y is non-empty;
<X,Y> ε f(R3) if and only if the intersection of X and Y is empty;
<X,Y> ε f(R4) if and only if X and the complement of Y is non-empty;
Then, letting f be an interpretation, and letting X and Y be terms (which we assume here have been defined), we would have:
"All X are Y" is true under f if and only if <f(X), f(Y)> ε f(R1);
"Some X are Y" is true under f if and only if <f(X), f(Y)> ε f(R2);
"No X are Y" is true under f if and only if <f(X), f(Y)> ε f(R3); and
"All X are Y" is true under f if and only if <f(X), f(Y)> ε f(R4).

4.16. *Graphing standard form categorical syllogisms.*

4.16.1. A *standard form categorical syllogism* is a triple of standard form categorical sentences consisting of two premises and a conclusion, each of which contains two different terms, one of which is common to both premises and one of which is not, and where the two terms of the conclusion are those which are not common to both premises.
4.16.2. *Venn diagram graphs for standard form categorical syllogisms.* The usual graphical techniques to determine whether the conclusion of a categorical syllogism is a deductive consequence of its premises is to form three overlapping circles each representing one of the three terms of the syllogism in such a way that the graphical relationship between the circles in each pair represents the content of the two premises of the syllogism, and from which the content of the conclusion becomes thereby also represented.

4.17. *Truth condition of a standard form categorical sentence under an interpretation.* Let f be an interpretation of the variables "X," "Y," and "Z," let f^ and (G/f) be as defined above, and let VJ and VK be any two distinct variables from among "X," "Y," "Z." Let S be a standard form categorical sentence in "X," "Y," and "Z". Then:
(i) If S is of the form, "All VJ are VK, then S is true under f if and only if the septuple (G/f^)(<f(X), f(Y), f(Z)>) determined by f contains a 0 in those places corresponding to f(VJ) – f(VK)) (i.e., if and only if f(VJ) is graphically included in f(VK)).
(ii) If S is of the form, "No VJ are VK, then S is true under f if and only if the septuple (G/f^)(<f(X), f(Y), f(Z)>) determined by f contains a 0 in those places corresponding to the intersection of f(VJ) and f(VK) (i.e., if and only if f(VJ) is graphically disjoint from f(VK))
(iii) If S is of the form, "Some VJ are VK, then S is true under f if and only the septuple (G/f^)(<f(X), f(Y), f(Z)>) determined by f contains a 1 in at least one place corresponding to the intersection of VJ and VK.(i.e., if and only if the graphical intersection of f(VJ) and f(VK) is not empty).
(iv) If S is of the form, "Some VJ are not VK," then S is true under f if and only if the septuple (G/f^)(<f(X), f(Y), f(Z)>) determined by f contains a 1 in at least one place corresponding to the intersection of f(Vj) and the complement of f(VK) (i.e., if and only if the intersection of f(VJ) and the complement of f(VK) is not empty).

4.18. *Examples of truth conditions of particular standard form categorical sentences under interpretations.*
Let f be an interpretation of the variables "X," "Y," and "Z," let f^ and (G/f) be as defined above, and let S be a standard form categorical sentence. Then:
(i) If S is of the form, "All X are Y, then S is true under f if and only if the septuple (G/f^)(<f(X), f(Y), f(Z)>) determined by f contains a 0 as its first and fourth term (i.e., if and only if f(X) is graphically included in f(Y)).
(ii) If S is of the form, "No X are Y," then S is true under f if and only if the septuple (G/f^)(<f(X), f(Y), f(Z)>) determined by f contains a 0 as its second and fifth term (i.e., if and only if f(X) is graphically disjoint from f(Y)
(iii) If S is of the form, "Some X are Y," then S is true under f if and only the septuple (G/f^)(<f(X), f(Y), f(Z)>) determined by f contains a 1 as its second or fifth term (i.e., if and only if the graphical intersection of f(X) and f(Y) is not empty).

(iv) If S is of the form, “Some X are not Y,” then S is true under f if and only if the septuple (G/f^)(<f(X), f(Y), f(Z)>) determined by f contains a 1 in its first or fourth term (i.e., if and only if f(X) and the graphical complement of f(Y)) is not empty..
(v) If S is of the form, “Some X are Z,” then S is true under f if and only if the septuple (G/f^)(<f(X), f(Y), f(Z)>) determined by f contains a 1 in its fourth or fifth term (if and only if the intersection of f(Y) and f(Z) is not empty).
(vi) If S is of the form, “No Y are Z,” then S is true under f if and only if the septuple (G/f^)(<f(X), f(Y), f(Z)>) determined by f contains a 0 in its fourth and fifth place (i.e., if and only if f(Y) is graphically disjoint from f(Z)

4.19. *Graph based deduction on standard form categorical sentences.* The graph of a categorical sentence in standard form is the set of septuples determined by interpretations under which that sentence is true. The graph of a *set* of categorical sentences in standard form is the graphical intersection of the graphs of the sentences in that set. Given this definition of a graph of a standard form categorical sentence, we can state the positive and negative paradigms for such sentences as follows:

4.20. *Strong form of positive graphical paradigm for standard form categorical logic*: a given standard form categorical sentence is a deductive consequence of a set of categorical sentences if and only if the graphical intersection of the graphs of the categorical sentences in the set is graphically included in the graph of the given categorical sentence.

4.21. *Negative graphical paradigm for standard form categorical logic*: a given standard form categorical sentence is a deductive consequence of a set of standard form categorical sentences if and only if the graphical intersection of the graphs of the categorical sentences in the set is graphically disjoint from the graph of the negation of the given standard form categorical sentence.

4.22. We can summarize the content of 4.17 and 4.18 as follows: A set P of standard form categorical sentences deductively entails a given standard form categorical sentence C if and only if the graph of P is graphically included in the graph of C, or is graphically incompatible with the graph of the standard categorical formulation of the negation of C.

4.23. *Example of graph based deduction in standard form categorical logic.*
We show that the set consisting of the categorical sentences “All X are Y” and “Some X are Z” deductively entails the categorical sentence “Some Y are Z,” by noting (1) that the intersection of the graphs of the “All X are Y” and “Some X are Z”} is graphically included in the graph of the sentence “Some Y are Z,” or, equivalently, by noting (2) that the intersection of the graphs of the “All X are Y” and “Some X are Z” is graphically disjoint from graph of the negation, :”No Y are Z” of the sentence “Some Y are Z,”
<u>Proof of (1):</u> Let the general form of a denotation of “All X are Y” be <0 * * 0 * * *>, where * is either a 0 or 1, and the denotation of “Some X are Z” be <* * * 1 0 * *>. <* * * 0 1 * *>, or <* * * 1 1* *>. Note that the intersection of these two denotations is the set of those denotations which contain a 1 in the fifth place. Note that these are just the denotations which contain a 0 in the first and fourth places. Let the general form of a

denotation of “Some Y are Z” be <* * * 1 0 * *>, <* * * 0 1 * *>, or <* * * 1 1 * *> (i.e., those denotations which contain a 1 in either the fourth or fifth place. Let the general form of a denotation of “Some Y are Z” be <* * * * 1 0 *>, <* * * * 0 1 *>, or <* * * * 1 1 *> (i.e., those denotations which contain a 1 in either the fifth or sixth place).
The categorical sentences “All X are Y” and “Some X are Z” have as common denotations those of the following form: <* * * 0 1 * *>, which are clearly among the denotations of “Some Y are Z,” namely the denotations of the form: <* * * * 1 0 *> or <* * * * 1 1 *>. [18, 19]
<u>Proof of (2):</u> Let the general form of the intersection of the denotations of “All X are Y” and “Some X are Z,” namely the denotations which contain a 1 in the fifth place. Let the general form of the denotation of the negation “No X are Z” of “Some X are Z be <* * * 0 0 * *>, (i.e., the denotations which contain a 0 in both the fourth and fifth place), which is clearly incompatible with the set of denotations which contain a 1 in the fifth place.

Footnote 18. Note that by virtue of the * notation, we are able to describe these relationships in a compact way. For example, in full 0/1 notation the denotations indicated by
<0 * * 0 * * *>, say, would include: <0 0 0 0 0 0 0>, <0 0 1 0 * * *>, <0 0 1 0 0 0 0>,
<0 1 1 0 0 0 0>, ... , <0 0 0 0 1 0 0>, ... , <0 1 1 0 1 1 1>; that is, all seven term sequences of 0’s and 1’s which have a 0 in the first and fourth places. We note, further, that, since there are exactly five places where either a 0 or 1 could occur, there would be $2^5 = 32$ such denotations.

Footnote 19. There are $2^7 = 128$ different possible sequences f*(X), hence 128 different possible functions f*.

5. Some Preliminaries to Graph Based Machine Deduction on Natural Language Sentences

5.1. *Transition to natural language.* The discussion below regarding graph based machine deduction on natural language sentences parallels that presented above regarding graph based machine deduction on sentences of elementary algebra and categorical logic, but is novel in its details. We note that any effort to formulate graphical methods for parallel deduction needs to be based on precise notions of syntactic and semantic structure for sentences of natural language. In the cases of elementary algebra and categorical logic, we accepted the customary informal characterizations of their syntactic and semantic structures as given in the classroom. In the case of natural language, however, there exists no formal or informal characterization of syntactic and semantic structure which can be regarded as customary, that is, as widely accepted. Notions of syntactic and semantic structures for natural language forwarded in the literature are essentially designed for purposes[5] other than enabling graph based machine deduction.

5.2. *Procedure.* We list the components of *local graphs* for NL sentences in Section 5.4. below, but first briefly indicate the nature of the *local denotations* of NL sentences which those local graphs pictorially depict, and of the *global graph* of a given NL sentence as a linked array of the local graphs depicting the local denotations of that sentence. In Appendix C we indicate how *deductive connections* among given NL sentences are determined by specific relationships that hold among their global graphs.

5.3. *Finer breakdown of procedure.*
5.3.1. Local denotation of an NL sentence. A local denotation of an NL sentence relative to a given interpretation under which that sentence is true is a set whose structure corresponds to the conditions under which that sentence holds or fails to hold relative to that interpretation.
5.3.2. *Global denotation of an NL sentence.* The global denotation of an NL sentence is the set of its local denotations relative to *all permissible* interpretations under which that sentence is true, and would exhibit – in set theoretic terms – all possible circumstances under which that sentence holds or fails to hold.
5.3.3. *Local graph of an NL sentence.* A local graph of an NL sentence is a graph whose structure corresponds to the set theoretic structure of a local denotation of that sentence in the sense that that local graph and that local denotation are inter-retrievable.
5.3.4. *Global graph of an NL sentence.* The global graph of an NL sentence is a dot–linked[19] array of all the local graphs of that sentence.

Footnote 19. In the sense of "linked by dots," as described in Sections 5.4.1 and 5.4.2 below.

5.4. *Components of local graphs of NL sentences.*

5.4.1. Basic components

(i) *Nodes (•)* depict entities in the domain of discourse.

(ii) *Arrows* (———>) depict unary or binary relations on elements in the domain of discourse;

(iii) *Dotted lines* (……..) depict the identity relation among entities in the domain of discourse.

(iv) *Barred arrows* (——|——>) depict the complement of the relation depicted by the unbarred arrow on which the bar is superimposed;

(v) *Barred dotted lines* (….|….) depict the complement of the identity relation.

5.4.2. Paths and operations on paths

(vi) *Arrow paths* (•———>•) , (•——|——>•) , (•———>) , (•——|——>) ,

(———>•), and (——|——>•) depict that the entities depicted by the nodes the arrows stand in the binary or unary relation depicted by the arrows; (placement of a node at the origin or terminus of an arrow signifies respectively that the element depicted by the node is in the domain or range of the relation depicted by the arrow;

(vii) *Unbarred dot paths connecting nodes* depict that the entitiies depicted by those are identical.

(viii) *Barred dot paths connecting nodes* depict that the entities depicted by those are not identical.

(ix) *Unbarred dot paths connecting unbarred arrows or connecting barred arrows* depict that the relations depicted by those arrows are identical [20]***;***

(x) *Unbarred dot paths connecting an unbarred and a barred arrow* depict that the relations depicted by those arrows are complementary [20]***;***

(xi) *Dashed lines* (**--------)** are used to connect two or more arrow or dot paths to depict the conjunction of the relations they depict. (See Appendix F for examples.)

(xii) *Solid Braces* (| | |) are syncategorematic elements composed of two or more vertical lines topped by an unbroken horizontal line, and are used to connect two or more arrow or dot paths to depict n-term relations for n > 2; the brace is barred if there is a bar on any part of the horizontal line; if the brace is unbarred, it depicts the *composition* of the relations which the connected arrow or dot paths depict; if the brace is barred, it depicts the complement of the composition of those relations. (see Appendix F for examples).

(xiii) *Broken Braces* (| | |) are syncategorematic elements composed of two or more vertical lines topped by a horizontal line consisting of dashes, and are used to connect two or more arrow or dot paths to depict the conjunction of the relations they depict. If the brace is barred, it depicts the complement of that conjunction. The primary use of broken braces is that they provide a base on which to place a bar to thereby depict the *complement* of the relations depicted by the arrow or dot paths they connect. (Without the bar, the broken brace depicts the same conjunction as that depicted by the connection.)(see Appendix F for examples).

Footnote 20. Dots used in this capacity have the meaning that their underlying relations are identical. They have the same meaning as they have when used to connect elements in local graphs, namely as the identity relation.

6. Graphical Representation of "Every man loves some woman."

6.1. *The sample sentence: "Every man loves some woman."* For simplicity and comprehensibility, we illustrate some of the underlying notions of natural language syntactic and semantic structure as they apply to the sentence, “Every man loves some woman,” which is simple enough to serve as an intuitive base for indicating what is involved in transitioning from the character string that comprises that sentence to its syntactic structure, then to its semantic structure and, finally, to its graphical structure suited to near-instantaneous deductive determinations by machines.

6.2. *Syntactic Structure of “Every man loves some woman.”* A characterization of the syntactic structure of this sentence involves the identification of those (not necessarily contiguous) subsequences of the character strings comprising this sentence which are “meaning bearing,” that is, to which a set theoretic meaning can be assigned under a semantic characterization of this sentence.

6.3.. *Semantic Structure of “Every man loves some woman.”* A characterization of the semantic structure of this sentence involves the identification of the meanings that are to be assigned to its meaning bearing substrings, namely, the meanings of “every,” “man,” “loves,” “some,” “woman,” “every man,” “some woman,” and “Every man loves some woman.” [21, 22]

Footnote 21. When formalized, these subsequences of character strings are respectively replaced by representational morphemes UN, MAN, LOVES, SOME, and WOMAN. See Section 8, below.

Footnote 22. In this example, the substrings comprising these expressions are displayed as complete, for simplicity of exposition. In a fuller development, certain of them would be exhibited as composed of special sub-expressions which are implicit in them, such as those constituting a plurality morpheme applied to “man” to form “men,” a present tense morpheme applied to a tenseless form “love’ to form “loves.” That fuller development is described in an umpublished manuscript [22].

6.4. *Meanings of meaning bearing substrings of “Every man loves some woman.”* For simplicity in describing set theoretic meanings that are to be assigned to the meaning bearing substrings of this sentence, we consider a simple interpretation which assigns, to “man,” the set {m1, m2} consisting of two men, namely m1 and m2, and to “woman,” the set {w1, w2}, consisting of two women, namely w1 and w2, and which assigns, to “loves” the two-place relation among objects in the domain of discourse such that the first named object in this relation loves the second named object. The remaining meaning bearing substrings in this sentence are “all” and “some,” which are assigned – as their meanings – certain functions which convert any set to which they are applied to a set of subsets of that set. Thus the function assigned to “all,” when applied to the meaning of “men,” i.e., when applied to the set {m1, m2}, converts it to the set {{m1, m2}}. And the function assigned to “some,” when applied to the meaning of “woman,” i.e., when applied to the set {w1, w2}, converts it to the set {[w1, w2}, {w1,}, {w2}} i.e., converts it to the set of all non-empty subsets of the set {w1, w2}.[23]

Footnote 23. These ideas and constructions are extended in [22] to apply to a wide range of quantifiers, determiners, and modifiers.

6.5. *Graphical Structure of “Every man loves some woman.”* The graphical structure of this sentence involves the association of four nodes representing the objects m1, m2, w1, and w2, drawn in such a way that the nodes representing the pair m1 and m2, and the nodes representing the pair w1 and w2 are each arranged in a vertical column, as follows:

Figure 2

6.6. *Use of barred and unbarred arrows.* The relation “loves” holding between the man mi and the woman wj, $1 \leq i, j \leq 2$, would be represented in this graph by imposing an *unbarred* arrow originating at the node labeled mi and terminating at the node labeled wj, and the relation “loves” not holding between a man mi and a woman wj, $1 \leq i, j \leq 2$, is represented in this graph by a *barred* arrow originating at the node labeled mi and terminating at the node labeled wj; thus unbarred arrows represent the relation “loving,” and barred arrows represent the relation, “not loving.”

6.7. *Range of possible graphs.* Among the sixteen possible graphs that can be drawn on such pairs of nodes using similarly oriented unbarred and barred arrows joining them (shown in Figure 3 below), only nine of these graphs could be associated with the relation expressed by the sentence “Every man loves some woman” taken relative to a binary relation r^2 which includes that relation. These are the graphs: (1) – (5), (7) – (10).

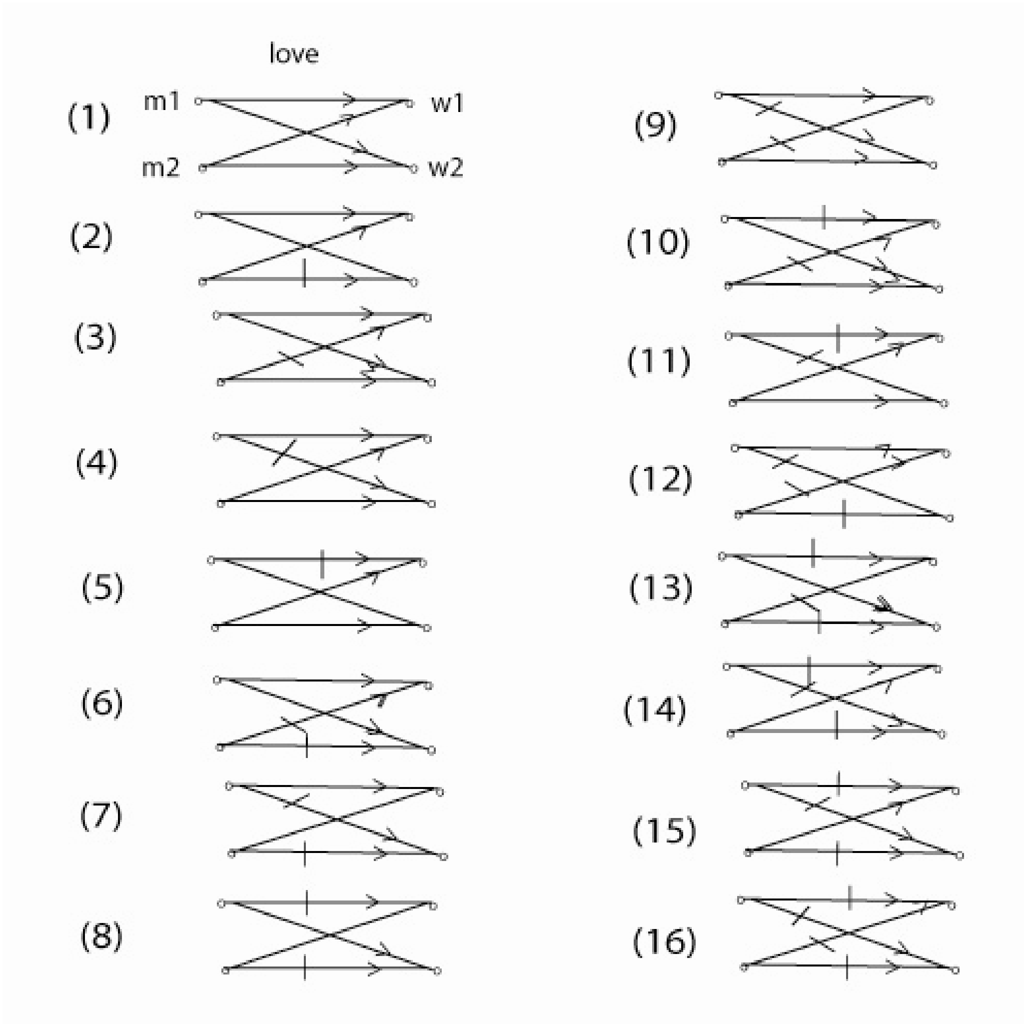


Figure 3

6.8. We illustrate how one such graph, namely, graph (2) (the second on the left in Figure 3), would be associated with the relation expressed by this sentence, when taken relative to the following binary relation r^2:

(a) Let r^2 = {<m1, w1>, <m1, w2>, <m2, w1>, <w1, m1>, <w2, m1>, <x, y>, <z, v>},

where x, y, z, and v are entities other than men and women in the domain of discourse between which the relation of loving also holds.

The set (b) below is that subset of the relation (a) restricted to men loving women (i..e., as opposed to men loving anything other than women or women loving anything), and is obtained as the intersection of the set (a) with the Cartesian product:
{m1, m2} x {w1, w2}. This intersection is (b) below:

(b) {<m1, w1>, <m1, w2>, <m2, w1>}

The particular relation (b) and its graph (graph (2) in Figure 3, above) are determined by the binary relation r^2, that is, by the binary relation (a). We have noted above that there are exactly nine subsets of the Cartesian product, {m1, m2} x {w1, w2} (and their associated complements relative to that Cartesian product) that can be obtained by intersecting that Cartesian product with binary relations like the relation (2) on the domain of discourse, each such binary relation r^2 determining a particular subset of that Cartesian product. In particular, the relation (b) was determined by intersecting that Cartesian product with the binary relation (a).

6.9. *Relations associated with the local graphs (1), (2),(3), (4), (5), (7),(8)),(9), (10)* It can be easily verified that the nine relations obtained in this way are respectively associated with the local graphs (1), (2), (3), (4), (5), (7), (8), (9), and (10), of the sentence, "Every man loves some woman," shown in Figure 3 are R1 through R9 below, and that these nine relations represent all possible ways that the sentence, "Every man loves some woman," can be true under the above assumptions. In each of these nine local graphs, the nodes on the left depict the set of men {m1, m2}, the nodes on the right depict the set of women {w1, w2}, the unbarred arrows depict the relation R of loving, and the barred arrows depict the relation Not-R of not-loving. We represent the empty set by the expression,"θ", and represent the relative complement of the relation Ri by the expression, "Not-Ri." With these understandings, the nine relations are as follows:

R_1 = {<m1, w1>, <m1, w2>, <m2, w1>, <m2, w2>},
Not-R_1 = θ;

R_2 = {<m1, w1>, <m1, w2>, <m2, w1>},
Not-R_2 = {<m2, w2>};

R_3 = {<m1, w1>, <m1, w2>, <m2, w2>},
Not-R_3 = {<m2, w1>};

R_4 = {<m1, w1>, <m2, w1>, <m2, w2>},
Not-R_4 = {<m1, w2>};

R_5 = {<m1, w2>, <m2, w1>, <m2, w2>},
Not-R_5 = {<m1, w1>};

R_6 = {<m1, w1>, <m2, w1>},
Not-R_6 = {<m1, w2>, <m2, w2>}};

R_7 = {<m1, w2>, <m2, w2>},
Not-R_7 = <m1, w1>, <m2, w1>};

R_8 = {<m1, w1>, <m2, w2>},

Not-R_8 = {<m1, w2>, <m2, w1>};

R_9 = {<m1, w2>, <m2, w1>},
Not-R_9 = {<m1, w1>, <m2, w2>}.

6.10. *Local denotations depicted by local graphs (1), (2), (3), (4), (5), (7), (8), (9), (10).*
Let f_1 , ..., f_9 be interpretations such that f_i (LOVES) = Ri, for $1 \leq i \leq 9$.
Then the *local denotations* of "All men love some woman" relative to the interpretations f_1 , ..., f_9 are the following sets, and the *global denotation* of this sentence is the set consisting of these nine local denotations. [24, 25]

{{ f_1 (LOVES), {<m1, w1>, <m1, w2>, <m2, w1>, <m2, w2>}} U
{{f_1 (LOVES)^c , θ}};

{{ f_2 (LOVES), {<m1, w1>, <m1, w2>, <m2, w1>}} U
{{f_2 (LOVES)^c , <m2, w2>}};

{{ f_3 (LOVES), {<m1, w1>, <m1, w2>, <m2, w2>}} U
{{f_3 (LOVES)^c ,<m2, w1> }};

{{ f_4 (LOVES), {<m1, w1>, <m2, w1>, <m2, w2>}} U
{{f_4 (LOVES)^c ,<m1, w2> }};

{{ f_5 (LOVES), {<m1, w2>, <m2, w1>, <m2, w2>}} U
{{f_5 (LOVES)^c ,<m1, w1> }};

{{ f_7 (LOVES), {<m1, w1>, <m2, w1>}} U
{{f_7 (LOVES)^c ,<m1, w2> , <m2, w2> }};

{{ f_8 (LOVES), {<m1, w2>, <m2, w1>}} U
{{f_8 (LOVES)^c ,<m1, w1> <m2, w2>}}.

{{ f_9 (LOVES), {<m1, w1>, <m2, w2>}} U
{{f_9 (LOVES)^c ,<m1, w2> <m2, w1>}};

{{ f_{10} (LOVES), {<m1, w2>, <m2, w2>}} U
{{f_{10} (LOVES)^c ,<m1, w1> <m2, w1>}};

Footnote 24. While in a certain sense the addition of references to the relations , as in f_1 (LOVES), appears redundant inasmuch as it is "understood" in its context, we need to include explicit references to the relations and their complements for transitioning from local denotations to their graphical depictions as local graphs.

Footnote 25. Note that each of these local denotations is inter-retrivable from the local graph which it depicts.

Finally, the graphs, (1), (2), (3), (4), (5), (7), (8), (9), (10) are the *local graphs* of "Every man loves some woman" relative to the interpretations f_1 , ..., f_9 , respectively, and the *global graph* of this sentence relative to those interpretations is a linked[26] array of all these local graphs.

Note that local and global denotations are sets and that local and global graphs are graphs. The point of this paper is that near-instantaneous machine determinations of deductive connections among NL sentences can be more readily executed on suitable graphs of their denotations rather than on the denotations themselves.

Footnote 26. The links joining the local graphs is this array are dotted lines, as described in Section 9.1. below, and exhibited later in Appendix G.

7. Generalization to Arbitrary NL Sentences

We consider the construction described in the preceding Section 6 in a more general setting.

7.1. *Syntactic representations.* As remarked earlier, a semantic approach to graph based deduction on NL sentences needs to be based on a precise characterization of NL syntactic and semantic structure. In this paper we sketch these structures in outline.[27]

Footnote 27. A more complete characterization is given in [22].

7.1.1. *Relation expressions.* A *relation expression* is a syntactic representation of a character string which, in a given occurrence, is semantically interpreted as an m-place relation which holds among given m-tuples of elements of the universe of discourse. Relation expressions also include (syntactic representations of) the logical connectives.
7.1.2. *Thing-expressions.* A *thing-expression* is a syntactic representation of a character string which is semantically interpreted as a set of subsets of the universe of discourse. We refer to such a set as a "thing."
7.1.3. *Modifier-expressions.* A *modifier-expression* is a syntactic representation of a character string which, in a given occurrence, is semantically interpreted as a function which maps a relation-expression or thing-expression onto either a thing expression or relation expression. We refer to such a function as "modifier."
7.1.4. *NL Sentences*. An *NL sentence S* can be schematically indicated as an n+1 term sequence $<r, t_1, ..., t_n>$, for some positive integer n, where r is an n-term relation expression and $<t_1, ..., t_n>$ is a sequence of thing expressions $t_1, ..., t_n$.
7.1.5. *Representational morphemes.* A representational morpheme is, roughly, the smallest semantically interpretable unit that enters into the syntactic representation of an NL sentence. While corresponding to "morphemes" as usually understood, they differ in that they are semantically interpretable whereas "morphemes" in the usual sense may not be.

7.1.6. *Representational Compounds.* A representational compound is a representational morpheme to which one or more modifiers are attached.

7.2. *NL Interpretations.* An *NL interpretation f* is a function which assigns an n-term relation to every relation expression r and assigns a set of elements of the domain of discourse to every thing expression t. The only *variables* in our treatment of NL are the relation expressions. *Thus an NL interpretation f is wholly determined by the relations it assigns to relation expressions.* All other meaning bearing expressions in S are *constants*, that is, expressions which are assigned the same meaning by every NL interpretation.[28]

Footnote 28. This is quite different form the use of constants in elementary algebra and in categorical logic, but the function they have here in our treatment of NL sentences is wholly analogous to the functions that constants play in elementary algebra and categorical logic, namely to distinguish those meaning bearing expressions which are assigned te same meanings in all interpretations from those meaning bearing expressions which are not.

7.3. *Local denotations of NL sentences.* The *local denotation of an NL sentence* *S = <r,* t_1*, ...,* t_n *> relative to a given NL interpretation f* will be defined as a union of two sets A, B, of n+1- term sequences each such that, (i) f(r) is the first term of all the sequences in A, and f(not-r) is the first term of all the sequences in B, and (ii) the sequence of each of the remaining n terms of A is an element of f(r), and the sequence of each of the remaining n terms of B is an element of f(not-r). [29]

Footnote 29. This notion is spelled out in more detail in Appendix C, particularly in Section C.3, where it is applied to our sample sentence in some detail.

8. Example. Graph Based Deduction on "Every man loves some woman. First Approximation.

8.1. *Syntactic Representation of "Every man loves some woman,"* The components which enter into the syntactic representation of this sentence are as follows: The pre-formalized English versions of these components are the character strings, taken in order of their occurrence, which are: "every," "man," "loves," "some," and "woman," and these are respectively formalized as the following strings of semantically interpretable representational morphemes: "UN," "MAN," LOVES," "A," "D," "INDEF," "WOMAN." The morphemes "UN," "A", "D," and "INDEF" are modifier expressions, "MAN" and "WOMAN" are thing expressions, "LOVES" is a relation expression. The expressions "UN MAN," "INDEF WOMAN," "LOVES A," and "LOVES A D" are representational compounds. We can provisionally syntactically represent the full sentence as: [UN MAN] [[[LOVE] A] D] [[INDEF [WOMAN]] [30]

Footnote 30. This is a simplified version of the syntactic representation of this sentence. See Appendix F.

8.2. *Semantic interpretation of "Every man loves some woman."* The representational morphemes and representational compounds identified in Section 8.1. above are semantically interpretable as follows: the representational morphemes “MAN” and “WOMAN’ are interpreted as subsets of the universe of discourse, the representational morpheme “UN” is interpreted as a modifier (i.e., a function) which, in application to the representational morpheme “MAN” is interpreted as the singleton set whose only member is the interpretation of “MAN.” The representational morpheme “LOVES” is interpreted as a 0 place relation; the representational compound “LOVES A” is interpreted as a one place relation (whose first place, intuitively, is to be occupied by some entity that loves); the representational compound “LOVES A D” is interpreted as a two place relation (whose first place, intuitively, is to be occupied by some entity that loves and whose second place by some entity that is loved by the entity occupying the first place; intuitively “A” signifies the agent of the relation, and “D” signifies the direct object of the relation). “INDEF” is interpreted as a modifier which, in application to the representational morpheme “WOMAN,” is interpreted as the set of all subsets of the interpretation of “WOMAN,” (For simplicity, further representational morphemes interpreted as indicating number and tense, are suppressed, and are not treated in this paper[31] . Notational devices allowing the association of thing expressions with case morphemes independently of the order in which those case morphemes occur in the relation expression are introduced later in this paper, as are notational devices signifying the order in which thing expressions are to be considered independently of the order in which they occur in the sentence. These two types of devices enable a wide range of possible readings to be assigned to a given character string in written form, which are ordinarily rendered in oral speech by intonation, stress, pauses, and so on.

Footnote 31. Number and tense are treated in [22].

8.3. *Figure 3 reviewed:* For definiteness and simplicity, we have been assuming in our above discussion of the sentence, “Every man loves some woman,” that the universe of discourse D consists of all human beings, and that the elements man_1 and man_2 are all the *men* in D, and that the elements $woman_1$ and $woman_2$ are all the *women* in D. Let us assume further that the relation we are concerned to represent is that of *loving* as it pertains to men loving women, that we can represent the circumstance that man_i loves $woman_j$, for $1\leq i, j \leq 2$, by a diagram consisting of an unbarred arrow joining the node representing man_i to the node representing $woman_j$, and that we can represent the circumstance that man_i *does not* loves $woman_j$, for $1\leq i, j \leq 2$, by a diagram consisting of a *barred* arrow joining the node representing man_i to the node representing $woman_j$.

8.4. *Use of dotted lines joining local graphs in Figure 3.* Let ID be a representational morpheme which is semantically interpreted as the identity relation, and which we graph (here, as earlier) as a dotted line joining other graphical elements. Specifically, dotted lines linking nodes represent that they stand for the same entity; dotted lines linking two unbarred or linking two unbarred arrows represent that they stand for the same relation;, and that dotted lines linking an unbarred with a barred arrow represent that they stand for complementary relations. Figure 3 above would then represent all 16 possible relations of loving as it pertains to man_1, man_2 and $woman_1$, $woman_2$.

9. Example. Graph Based Deduction on "Every man loves some woman." Second Approximation.

9.1. *Global graphs as representations of sentence meanings.* For any subset S of individual diagrams (1) - (16) in Figure 3, let g(S) be the *linked array* of the diagrams in S, where links are indicated by dotted lines. Each diagram in Figure 3 is a local graph and the linked array of all diagrams (local graphs) in S is a *global graph*. Thus S is a set of local graphs whereas g(S) is the global graph composed as a linked array of the local graphs in S. Let DIAG be the linked array of all diagrams (1) – (16). Consider the set K of all sentences that can be formed as sentential combinations of sentences of the form "(Q1) men loves (Q2) women," where Q1 and Q2 are quantifiers chosen from among "all," "some," "no," "any," "at least two," "at most two," "at most one," "exactly one," etc. It is fairly obvious that, for any sentence k in K there is a linked sub-array k^ of DIAG which graphically represents the meaning of k. For example, referring to the numbering used in Figure 3, and taking k_{16} as the sentence, "Every man loves all women," the linked sub-array k_1^ of S = {(16)} graphically represents the meaning of k_{16}; if k_2 is the sentence, "Every man loves some women," the linked sub-array k_2^ of DIAG = {(1), (2), (3), (4), (5), (7), (8), (9), (11)}represents the meaning of k_2; if k_3 is the sentence "At least one man loves at least two women," the linked sub-array k_3^ of DIAG = {(1), (2), (3), (4), (5), (6), (10)} represents the meaning of k_3; if k_4 is the sentence, "Some men love no women," the linked sub-array k_4^ of DIAG = {(6),(11), (12), (13), (14), (15), (16)} represents the meaning of k_4; if k_5 is the sentence, "Some men love some women," the linked sub-array k_5^ of DIAG = {(1), (2), (3), (4), (5), (6), (7), (8), (9), (10), (11), (12), (13) (14), (15)}represents the meaning of k_5.

9.2. *Deductive relations and global denotations of sentences in Figure 3.* Let Den_f(k) be the local denotation of the sentence k under the interpretation f, and let Den(k) be the global denotation of k that is, let Den(k) be the set of all local denotations k/f, as f ranges over all permissible interpretations. Deductive relationships among the sentences of Figure 3 resolve to set theoretic inclusion relations on the global denotations of those sentences. That is, a sentence k_j is deducible from a sentence k_i if and only if the global denotation Den(k_i) of k_i is set theoretically included in the global denotation Den(k_j) of k_j .; and a sentence k_j is deducible from a set { $k_{i1, \ldots,}$ k_{in} } if and only if the intersection of the global denotations of the sets $k_{i1, ...,.,}$ k_{in} is (set theoretically) included in the global denotation of the set k_j.

9.3. *Deductive relations and global graphs.* A *global graph V is a subgraph of a global graph W if and only if all local graphs in V are also local graphs in W.* If k is a sentence, let G(k) be the global graph of k (i.e., the linked array of all local graphs of k). Deductive relations among the sentences of Section 8.1 resolve to graphical inclusion relations (in the above sense) on the global graphs of those sentences as follows: Letting DIAG be the linked array of all diagrams (1) – (16) as stated in Section 7.7.1.,we have that a sentence k_j is deducible from a sentence k_i if and only if the global graph G(k_i) of k_i is a subgraph of the global graph G(k_j) of k_j ; and a sentence k_j is deducible from a set { $k_{i1, \ldots,}$ k_{in} } if and only if the intersection of the global graphs of $k_{i1, ...,.,}$ k_{in} is graphically included in the global graph of k_j. For example, the deducibility of each of sentences k_2 , k_3 and k_5 from sentence k_1 corresponds, respectively, to the graphical inclusion of each of $G(k_2)$, $G(k_3)$ and $G(k_5)$ in $G(k_1)$. Moreover, the determination that any given linked sub-array of DIAG is included in another linked sub-array of DIAG *can be executed in parallel*, inasmuch as all such determinations are independent of each other. In this sense, This is exactly comparable to the familiar circumstance from classroom algebra whereby a given equation or inequality e is deducible from a given system of equations and inequalities if and only if the intersection of the solution sets of the equations and inequalities in the set is set theoretically included in the solution set of the given equation or inequality.

9.4. These are the simplest types of deductive connections among global graphs. Their simplicity derives from the fact that their constituent local graphs are *similar* in the sense that they differ at most in the pattern of bars on their connecting arrows. See Figures 4 and 5 for how the deducibility relation would be represented for simple types of *non-similar* local graphs. One example involves the deducibility of "John is not a man" (graphically represented by the graph at the bottom of Figure 4) from the pair of (premise) sentences, "John does not love Mary" and "Every man loves Mary" (represented by the linked graphs at the top of Figure 4). As another example, in Figure 5 we see the deducibility of "Mary is not Agnes" (graphically represented by the graph at the bottom of Figure 5) from the pair of (premise) sentences, "John loves Mary" and "John does not love Agnes." Here, as in Figure 4, the deducibility in question is represented graphically, not by the inclusion relation illustrated above, but by the circumstance that all graphical components of the graph representing the conclusion are linked to corresponding components of the linked graphs representing the two premises. We state without proof that all corresponding links can be identified in parallel.

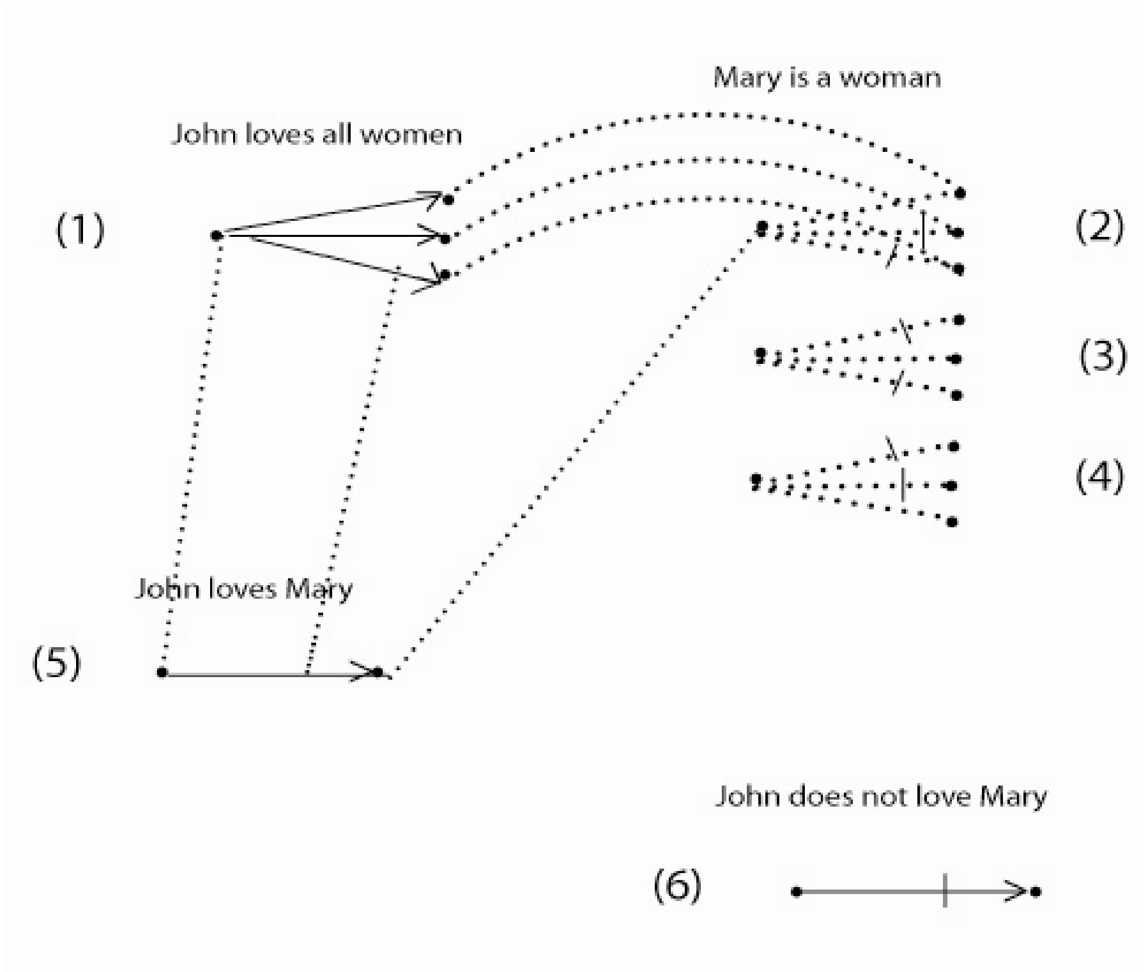


Figure 4

NOTE: For visual clarity we have omitted some of the dotted line links between similarly designated nodes.

COMMENT REGARDING FIGURE 4: The three local graphs of the two premise sentences of Figure 4, namely, "John loves all women" and "Mary is a woman," are the linked graphs <(1), (2)>, <(1), (3)>, and <(1), (4)>; and <(5)> is the single local graph of the conclusion sentence "John loves Mary" of Figure 4. <(6)> is the single local graph of the negation "John does not love Mary" of this conclusion sentence.

The conclusion "John loves Mary" can be established either directly by the linkages of its local graph <(5)> with each of the local graphs <(1), (2)>, <1), (3)>, and <(1), (4)> of the premises, or by the incompatibility of each of these local graphs of the premises with the negation of the intended conclusion as determined by their dotted line linkages.

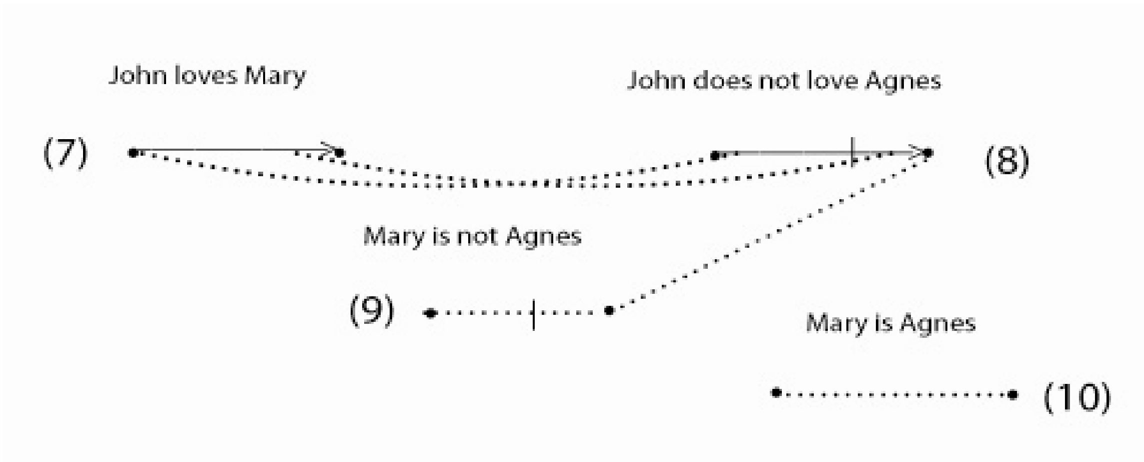


Figure 5

COMMENT REGARDING FIGURE 5: The one local graph of the two premise sentences of Figure 5, namely "John loves Mary" and "John does not love Agnes," is the linked graph <(7), (8)>; and <(9)>; is the single local graph of the conclusion sentence "Mary is not Agnes" of Figure 5. <(10)> is the single local graph of the negation "Mary is Agnes" of this conclusion sentence, which is incompatible with the local graph <(7), (8)> of the two premise sentences, as can be verified by joining similarly designated nodes with dotted lines. Note also that the conclusion sentence "Mary is not Agnes" cannot be established by joining similarly designated nodes, so that incompatibility of the local graph of the premises with the negation of the intended conclusion is the only way to establish that conclusion.

10. Generalizing Semantic Notions Relating to Figure 3

10.1. *Correspondences between English sentences and their representing graphs*. The correspondences between English sentences and their representing graphs alluded to above are intuitive, and can essentially be “read off” their representing graphs on the basis of the meanings of the graphical components of those graphs. *We now need to establish these correspondences in terms of correspondences between the syntactic representations of these sentences and their graphical representations as illustrated in Figure 3 on more rigorous semantic grounds.* For this purpose we need to indicate the kind of syntactic sentence representations used for the sentences entering into Figure 3, as well as to elaborate on the underlying semantic and graphical notions which determine them.

10.2. *Interpretations of syntactic components.* We restate the notion of interpretation (as used above) explicitly *as a function* that assigns sets to syntactic components. Accordingly, let f be an interpretation which assigns a subset f(MAN) of the domain of discourse to the thing expression MAN , assigns a subset f(WOMAN) to the thing expression WOMAN , assigns a singleton set whose only member is f(MAN) to the thing expression [UN MAN], assigns the set of all non-empty subsets of f(WOMAN) to the thing expression [[INDEF [WOMAN]] and, finally, assigns a binary relation f([[LOVES] A] D]) on the universe of discourse to the relation expression [[LOVES] A] D].

10.3. *Permissible Interpretations.* In order to render meanings of expressions and their associated graphs sufficiently uniform to support the parallel execution of deductive operations on them, we impose certain restrictions on the interpretations that assign those meanings; interpretations satisfying those restrictions are referred to as "permissible interpretations." Roughly, two permissible interpretations f1 and f2 would be constrained to assign the same set to each thing expression, so that, for example, f1(MAN) = f2(MAN) and f1(WOMAN) = f2(WOMAN). And since the quantifiers UN and INDEF act as constants, i.e., have the same functional action applied to them by all interpretations, it follows that f1([UN MAN]) = f2([UN MAN], and f1([INDEF [WOMAN]) = f2([INDEF [WOMAN]). *Thus the only way that two interpretations f1 and f2 could differ would be in what they assign to relation expressions like [[LOVES] A] D]*

10.4. *Truth under Interpretations.* Letting Z be the syntactic sentence representation[32] [UN MAN] [[[LOVES] A] D] [[INDEF [WOMAN]] of the natural language sentence "Every man loves some woman," and letting f be an interpretation on Z, we define Z as *true under the interpretation f* if and only if there is a set P in f([UN MAN]) such that for all m in P there is a set C in f([INDEF [WOMAN]) such that, for all w in C, the pair <m, w> is an element in f([[[LOVES] A] D]).

Footnote 32. We note again that this sentence representation is in simplified linear form, and that the more accurate representation of this sentence is that given in tree form as described in Appendix E.

10.5. *Positive Profile of Z under the Interpretation f. The positive profile of Z under f, in symbols,* POS_f (Z), is the intersection of the relation f([[[LOVES] A] D]) with the Cartesian Product f(MAN) x f(WOMAN), that is, POS_f (Z) is the set of all pairs <m, w> such that m is an element of f(MAN), w is an element of f(WOMAN) and <m,w> ε f(LOVES). *The negative profile of Z under f, in symbols,* NEG_f (Z), is the intersection of the complement of the relation f([[[LOVES] A] D]) with the Cartesian Product f(MAN) x f(WOMAN), that is, NEG_f (Z) is the set of all pairs <m, w> such that m is an element of f(MAN), w is an element of f(WOMAN) and m does not loves w.

10.6. *Chain Functions.* Letting X be the pair <f([UN MAN_T]), f([INDEF [$WOMAN_t$]])>. we define a *chain function through X* to be a function g such that, for every element y belonging to a member set of f([UN MAN]), g(y) is a member set of f([INDEF [WOMAN]).

10.7. *Traces.* Letting X and g be as above, we define *the trace of g through X* to be the set of all pairs <m, w> such that m is an element of a member set of f([UN MAN]), and w is an element of g(y). (Note: There are many possible chain functions g through X but exactly one chain function whose trace is identical with the positive profile of Z under f if Z is true under f.)

10.8. *Correspondence between the Pairs <m, w> in $Pos_f(Z)$ and Un-negated Arrows in Diagrams of Figure 3.* For every diagram d in Figure 3, the array of all un-negated arrows v occurring in d is a direct graphical depiction of the pairs. <m, w> in $\text{Pos}_f(Z)$, where the element m corresponds to the initial node of v and w corresponds to the terminal node of v.

10.9. *Correspondence between the Pairs <m, w> in $NEG_f(Z)$ and Negated (Barred) Arrows in Diagrams of Figure 3.* For every diagram d in Figure 3, the array of all negated arrows v occurring in d is a direct graphical depiction of the pairs. <m, w> in $\text{Neg}_f(Z)$, where the element m corresponds to the initial node of v and w corresponds to the terminal node of v.

10.10. *Correspondence between X and Diagrams of Figure 3:* Let Z, f, and X be as above. Then for every trace t through X, we associate the unique diagram from among those in Figure 3 each of whose unnegated arrows corresponds a unique pair <m, w> in t.

10.11. We note that there are exactly nine different relations f([[[LOVES] A] D]) restricted to the Cartesian product f(MAN) x f(WOMAN), where f (MAN) = {m1, m2} and f(WOMAN) = {w1, w2}woman), that is, nine different relations *Pos_f(*[UN MAN] [[[LOVES] A] D] [[INDEF [WOMAN]]*)* The graph corresponding to this particular relation is local graph (10) of figure 3.

Appendix A. Basic Ideas: A Summary

A.1. *Interpretations, Denotations, and Truth.*
An *interpretation* for a language L is a function which recursively assigns a *denotation* to every meaningful expression of that language, ultimately assigning denotations to sentences of that language in terms of the denotations of their meaningful parts. All denotations are sets. The denotation of a sentence relative to a given interpretation is a non-empty set only if it satisfies a *defining condition* particular to that sentence and that interpretation. That defining condition is stated in the language of set theory, which is the semantic meta-language for L, and is stated in set-theoretic terms that specify the relationship which needs to hold among the denotations of the meaningful parts of that sentence. If the defining condition holds in set theory (by virtue of the denotations assigned to those meaningful parts by that interpretation) then that sentence is regarded as *true under that interpretation*; *otherwise it is false under that* interpretation. A *set of sentences* has a denotation relative to an interpretation, namely the set of all denotations of its member sentences, provided that all its member sentences are true under that interpretation, in which case we say that the set is *true under that interpretation.*; *otherwise* the denotation of that premise set is the empty set relative to that denotation, and we say that *the premise set is false under that interpretation*. Finally, *the denotation of a sentence* is the set of its denotations relative to *all* interpretations for the language

under which it is true, and - in a similar fashion - *the denotation of a premise set* is the set of its denotations relative to *all* interpretations for the language under which all its member sentences are simultaneously true.

A.2. *Graphs.* The graphical apparatus we propose is designed to enable a machine to *at least approximate deductive connections that hold among sentences that are suitably graphically represented.* There are numerous ways in which deductive connections can be identified graphically. The graphical apparatus we propose has the advantage that it is applicable to a very wide range of NL sentences, and can be executed simultaneously on sets of sentences of arbitrary size. We distinguish between local and global denotations of a sentence: the *local denotation of a sentence* is the denotation of that sentence relative to a given interpretation under which that sentence is true; the *global denotation of a sentence* is the set of all its local denotations. We analogously distinguish between local and global graphs of a sentence: the *local graph of a sentence* is a graphical depiction of a given local denotation of that sentence with which it is inter-retrievable, and the *global graph of a sentence* is a linked array of its local graphs, and is inter-retrievable with its global denotation.

A.3. *Graph-based deduction.* The circumstance that a given set of sentences deductively entails a given sentence, which is characterized in the customary (model theoretic) way as holding just in case the given sentence is true under every interpretation under which all sentences in the set are true, is re-characterized in graphical terms as holding just in case every join among the mutually consistent local graphs of the sentences in the set graphically includes a local graph of the given sentence (Weak positive deductive paradigm).

A.4. *Advantage of proposed graphical approach.* There are numerous ways in which deductive relationships can be identified graphically. The advantage of our graphical approach is that it is grounded in model theoretic semantics in such a way that it is applicable to a very wide range of NL sentences, and is such that machine operations on graphs used to make deductive determinations can be executed *simultaneously* and *globally*, that is, *executable on sets of sentences of arbitrary size without regard to their potential relevance to making those deductive determinations.* The relative simplicity and directness of graphical deduction can be appreciated by comparing the entailment diagrams exhibited in Figures 3, 4, 5, and below in Appendix I using local graphs of sentences with what would be required in operating directly with their set theoretic denotations as defined in Appendix C.3 below.

Appendix B. Syntax of NL Sentences: A Summary

We need to define syntactic structures for natural language which can, in turn, support the definition of denotations which can then be depicted as graphs on which deductive *determinations can be near-instantaneously executed.* We indicate the intended denotations and then describe in summary form the syntactic structures which support them.

B.1. *Three types of NL expressions.* We distinguish three classes of expressions in terms of the kinds of entities they denote under interpretations:

B.1.1. *Relation expressions.* A relation expression is a syntactic representation of a character string which, in a given occurrence, is regarded as denoting (relative to a given interpretation) an m-place relation which holds among given m-tuples of elements of the universe of discourse. Some examples of character strings which are syntactically represented as relation expressions in their typical occurrences: "---give---," "--- give --- to ---," "--- give --- to --- for ---, and so on, where dashes (---) stand for places to be occupied by noun phrases. Relation expressions also include (syntactic representations of) the logical connectives, ordinary conjunctions and, generally, any character string which is understood in a particular occurrence as denoting a relation. Relation expressions - when formalized - have the following *internal structure*: An *m-place relation expression* is composed of a *base relation* r (e.g., "give") together with m ordered *case expressions* (e.g., representing "agent," "direct object," "indirect object," and so on) each of which identifies the semantic role of one of the m thing-expressions which the relation r relates.

B.1.2. *Thing-expressions.* A thing-expression is a syntactic representation of a character string which, in a given occurrence, is regarded as denoting a "thing" (relative to an interpretation), where a "thing" is a set of subsets of the universe of discourse. Some examples of character strings which are syntactically represented as thing-expressions in their typical occurrences: "book," "the book," "every book," "some book," "at most two books," etc. Thing expressions also include complex constructions built up from other thing expressions, to include sentences as well which, when understood as thing-expressions, denote "events." Thing-expressions - when formalized - have the following internal structure: A thing-expression is composed of a base expression together with one or more one or more "modifier expressions," as described below.

B.1.3. *Modifier-expressions.* A modifier-expression is a syntactic representation of a character string which, in a given occurrence, is regarded as denoting a "modifier," that is, a function (relative to an interpretation) which maps a relation-expression or thing-expression onto either a thing or relation expression. Some examples of character strings which are syntactically represented as modifier-expressions in their typical occurrences: simple articles and adjectives, e.g., "the," "all," "tall," and phrases and clauses which in particular occurrences, change the denotation of a thing or relation expression to which they apply.

B.1.4. *Sentences.* A *sentence* is a syntactic representation of a character string which, in a given occurrence, is regarded as denoting an "event" or "state of affairs," can be schematically indicated in the form $r^n (a_1, ..., a_n)_{c.s}$, where r^n is an n-place relation expression composed of a base relation $(r^n)^B$ attached to which are zero or more modifiers, together with an *ordered set of zero or more case markers* $b_1, ..., b_m$ indicating the semantic roles to be played by each of the n thing expressions $a_1, ..., a_n$ occupying the n places in $r^n(a_1, ..., a_n)_{c,s,}$ The two functions c and s on the n thing expressions $a_1, ..., a_n$, impose orderings on those thing expressions which are respectively referred to as *the* *relative case ordering and the relative scope ordering on $a_1, ..., a_n$.* The *case function c* determines the association of the case markers $b_1, ..., b_m$ with the thing expressions $a_1, ..., a_n$. , in the sense that, for each i, $1 \leq i \leq n$, $c(a_i)$ is the order of that case among $b_1, ..., b_n$ which applies to the thing expression a_i. The *scope function s on $a_1, ..., a_n$* determines the scopes of each of the modifiers attached to r^n which govern each of $a_1, \ldots, a_n$ .The truth condition of the denotation of r^n $(a_1, ..., a_n)_{c,s}$ under an interpretation is regarded as describing an "event" or "state of affairs" to the effect that the denotations of the n thing expressions $a_1, \ldots, a_n$ relative to the two functions c and s stand in the relation denoted by r^n, For most sentences of English, case expressions are usually placed adjacent to the thing expressions they govern, and the case function is taken as the identity function; that is, it usually coincides with the order of occurrence of the thing expressions associated with given cases. But this is not the situation for all sentences of English, nor for sentences of many other languages. The syntactic structure of sentences must take into account each of these special orderings. For example, different relative case orderings correspond to the intuitive difference between "Every man loves some woman" and "Some woman loves every man," different relative scope orderings correspond to the intuitive difference between "Every man loves some woman" and "Some woman is such that every man loves her," and different combined relative case and relative scope orderings correspond to the intuitive difference between "Some woman loves every man" and "Some woman is loved by every man."

B.2. *Role of relative case and scope orderings.* For most sentences of English, case expressions are usually placed adjacent to the thing expressions they govern, while scope is usually indicated by the context in which thing expressions occur and by the manner in which they are stressed, which usually coincides with the order of occurrence of the thing expressions they govern. But this is not the situation for all sentences of English, nor for all sentences of many other languages. The syntactic structure of sentences must take into account each of these special orderings.

B. 2.1. *Purpose of appended relative orderings.* The purpose of appending the relative orderings c, and s to the simple schematic representation of an NL sentence is to simplify the association of syntactic representations to NL character strings, by maximally preserving the order of all constituent character strings in S with their order in the representation of S, a consequence which markedly simplifies the association of the syntactic structure of S with S. Put differently, the syntactic representation r^n $(a_1, ..., a_n)$ of an NL sentence S is incomplete, unless one assumes that the order of the thing expressions $a_1, ..., a_n$ in that representation is the same as their order of occurrence in S.

B.3. *Summary.* With these understandings, we schematically indicate the syntactic structure of a natural language sentence S as $r^n(a_1, \ldots, a_n)_{c,s}$ where: (1) r^n is the major n-place relation, (2) $a_1, \ldots, a_n$ are the n thing expressions of S, expressed here in the order of their occurrence in S, (3) c is the relative case ordering these n thing expressions in S, and (4) s is the relative scope ordering of S.

Appendix C. Semantics of NL Sentences: A Summary

C.1. *Positive and Negative Relational Profiles.* If a is a thing-expression, let a^B be *the base* of a, that is, the thing expression a stripped of all initial modifiers. Let f be an interpretation, let $r^m(a_1, \ldots, a_m)$ be a sentence, and let $CP(r^m(a_1, \ldots, a_m))$ be the Cartesian product $f(a_1{}^B)$ x… x $f(a_m{}^B)$. Finally, let $f(r^m)^c$ be the complement of the relation $f(r^m)$. Then we define the *positive relational profile of $r^m(a_1, \ldots, a_m)$ under f, which we write as POS_f $(r^m(a_1, \ldots, a_m))$, to be the intersection of the set $f(r^m)$ with $CP(r^m(a_1, \ldots, a_m))$*, and we define the *negative relational profile of $r^m(a_1, \ldots, a_m)$ under f, which we write as NEG_f $(r^m(a_1, \ldots, a_m))$, to be the intersection of the set $f(r^m)^c$ with $CP(r^m(a_1, \ldots, a_m))$.* [Analogy with elementary algebra of the plane with variables “x” and “y” is imperfect but instructive: Since there are no base expressions in any algebraic expression, $a_i{}^B$ is simply a_i , and the Cartesian product $f(a_1{}^B)$ x $f(a_2{}^B)$ becomes the pair <f(a_1), f(a_2)> ; the only relations r^m are the binary relations of equality and inequality and their negations, so that m = 2; f(a_1) and f(a_2) are real numbers built up out of the real numbers f(“x”) and f(“y”) using the customary algebraic operations of plus, times, exponentiation, etc.; the positive relational profile POS_f $(r^2(a_1, a_2))$ of $r^2(a_1, a_2)$ under f becomes the intersection of the set $f(r^2)$ with $CP(r^2(a_1, a_2))$, which is simply the pair <f(“x”), f(“y”)> if the result of respectively replacing “x” and “y” by f(‘x”) and f(“y”) in $r^2(a_1, a_2)$ is a true sentence of elementary algebra, and is the empty pair otherwise; and the negative relational profile NEG_f $(r^2(a_1, a_2))$ of $r^2(a_1, a_2)$ under f becomes the intersection of the set $f(r^2)^c$ with $CP(r^2(a_1, a_2))$, which is the pair <f(‘x”), f(“y”)> if the result of respectively replacing “x” and “y” by f(‘x”) and f(“y”) in $(r^2)^c(a_1, a_2)$ is a true sentence of elementary algebra, and is the empty pair otherwise.]

C.2. *Chain Functions and Traces:* Let f be an interpretation, and let $f(r^m)$, $f(a_1)$, …, $f(a_m)$, be denotations of r^m , a_1, …, a_m , respectively. We define a *chain function through the sequence ($f(a_1)$, …, $f(a_m)$)* as a function g which assigns, to every set $f(a_1)$, for $1 \leq i \leq m-1$, and for every element y belonging to $U(f(a_1))$, that is, belonging to any of the member sets of $f(a_1)$, a set g(i,y) belonging to one of the sets in $f(a_{i+1})$.[34] We note that this definition is proper on any sequence ($f(a_1)$, …, $f(a_m)$) of denotations of thing-expressions a_1, …, a_m relative to f. Let g be a chain function through the sequence ($f(a_1)$, …, $f(a_m)$). Then we define *the trace of g through ($f(a_1)$, …, $f(a_m)$)* as the set: $\{(z_1 , \ldots, z_m) \in D^m$ // for some $x_1 \in f(a_1)$, $z_1 \in x_1$, and $z_2 \in g(1, z_1)$, and $z_3 \in g(2, z_2)$ and … and $z_m \in g(m-1, z_{m-1})\}$. There are in general many possible chain functions through the sequence ($f(a_1)$, …, $f(a_m)$) of thing expressions of $r^m(a_1, \ldots, a_m)$ relative to f, but there is exactly one chain function whose trace is identical with the positive relational profile POS_f $(r^m(a_1, \ldots, a_m))$ of $r^m(a_1, \ldots, a_m)$ relative to f if $r^m(a_1, \ldots, a_m)$ is true under f, and there is no such chain function if $r^m(a_1, \ldots, a_m)$ fails to be true under f. [Continuing with the analogy with elementary algebra of the plane: Since $f(a_1)$, $f(a_2)$ are real numbers and, as such, have no member sets, and so the above definition of chain function is not proper on any sequence ($f(a_1)$, $f(a_2)$) of denotations of algebraic terms, so that the above definition of trace does not apply. On the other hand, we can define a trace as a degenerate notion whereby the trace of a sequence ($f(a_1)$, $f(a_2)$) of denotations of algebraic terms is simply that sequence.]

Footnote 34. We relativize the set g(i,y) to i in order to distinguish what g assigns to an occurrence of y in one set from what g assigns to an occurrence of y in another set..

C.3. *Local denotations of Sentences.* We define the local denotation $f(r^m(a_1, \ldots, a_m))$ of the sentence $r^m(a_1, \ldots, a_m)$ relative to the interpretation f, in symbols, $Den_f (r^m(a_1, \ldots, a_m)$, as the set:
$\{\{ <f(r^m), v> // v \in POS_f\ (r^m(a_1, \ldots, a_m))\}\} \cup$
$\{ <f(r^m)^c, v> // v \in NEG_f (r^m(a_1, \ldots, a_m)) >\}\}$,
if there is a chain function g through the sequence $(f(a_1), \ldots, f(a_m))$ such that the trace of g through $(f(a_1), \ldots, f(a_m))$ is identical with $POS_f (r^m(a_1, \ldots, a_m)))$; and is the empty set Θ, otherwise.[35, 36, 37]

Footnote 34. Defining the local denotation of a sentence in this manner seems to afford a fairly natural way to understand the meaning of that sentence and, in addition, renders this meaning in a form which can be directly graphically depicted. While there are other was of defining local denotations of sentences, it does not appear that other ways of defining them would afford as natural a way to both understand their meaning and be directly amenable to graphical depiction.

Footnote 35. Note that the denotation of the sentence $r^n(a_1, \ldots, a_n)c,s$ relative to an interpretation f is a singleton-singleton set if $POS_f(r^n(a_1, \ldots, a_n)c,s)$ is identical with the trace of some chain function through the sequence $<s(f(a_1)), \ldots, s(f(a_n))>$, and is the singleton of the empty set, in symbols, $\{\theta\}$, otherwise. The reason for having the denotation of a sentence be a singleton-singleton set is that we want to have sentences qualify also as thing-expressions, and the denotation of thing expressions are always sets of sets of elements of the universe of discourse. Note: (1) n-tuples of elements of the universe of discourse are also elements of the universe of discourse. (2) There are in general many possible chain functions on the sequence $(f(a_1), \ldots, f(a_n))$ of thing expressions of $r^n(a_1, \ldots, a_n)c,s$ relative to f, but there is exactly one chain function whose trace is identical with the profile of $r^n(a_1, \ldots, a_n)c,s$, relative to f if $r^n(a_1, \ldots, a_n)c,s$ is true under f, and no such chain function if $+^n(a_1, \ldots, a_n)c,s$ fails to be true under f. (3) The profile of $r^n(a_1, \ldots, a_n)c,s$ relative to an interpretation f is just that relation obtained by restricting of $f(r^n)$ to the domains $f(a_1), \ldots, f(a_n)$ ordered by s. The trace of a chain function g through $(f(a_1), \ldots, f(a_n))$ with respect to the ordering s, on the other hand, is an m-place relation on the domains $f(a_1), \ldots, f(a_n)$ ordered by s which is induced wholly by the set-theoretic structures of $f(a_1), \ldots, f(a_n)$. In order for $r^n(a_1, \ldots, a_n)c,s$ to be true it is necessary and sufficient that the profile and trace of $r^n(a_1, \ldots, a_n)c,s$, coincide. (4) The idea underlying the definition of the denotation of a sentence is that that denotation $f(r^n(a_1, \ldots, a_n)c,s$ is non-empty if and only if the structure imposed by the thing expressions of $r^n(a_1, \ldots, a_m)c,s$ is consistent with the structure imposed by the relation expression of $r^n(a_1, \ldots, a_n)c,s$.]]]

Footnote 36 The set $Den_f (r^m(a_1, \ldots, a_m)))$ can be completely graphically represented as a network of nodes and connecting arcs, where the un-negated connecting arcs graphically represent the relation and each m-tuple of the nodes they connect graphically represents m elements of the domain which stand in that relation, and where negated connecting arcs graphically represent the complement of the relation and each m-tuple of the nodes they connect graphically represent m elements of the domain which fail to stand in that relation.

C.4. *Graphs of local denotations.*The graph of a local denotation of a natural language sentence is a tree structure

C.5. *Global denotations of sentences.* We define the global denotation $Den (r^m(a_1, \ldots, a_m))$ of the sentence $r^m(a_1, \ldots, a_m)$ as the set of all non-empty local denotations $Den_f (r^m(a_1, \ldots, a_m)$ of $r^m(a_1, \ldots, a_m)$ such that f is a permissible interpretation of $r^m(a_1, \ldots, a_m)$.
C.4.1.Continuing the analogy with elementary algebra of the plane: (i) the definition of local denotation specializes as follows: Let r^2 be an equation or inequality; then the local denotation $Den_f (r^2(a_1, a_2)$ reduces to the union of the two sets (i) and (ii):: (i)$\{ <f(r^2), v>$ // v is a pair <f("x"), f("y")> such that, when the variables "x" and "y" are respectively replaced by the values f("x"), f("y") in $f(a_1)$ and $f(a_2)$, $f(a_1)$ and $f(a_2)$ stand in the relation $f(r^2)\}$ and (ii) $\{ <f((r^2)^c), v>$ // v is a pair <f("x"), f("y")> such that, when the variables

“x” and “y” when respectively replaced by the values f(“x”), f(“y”) in f(a_1) and f(a_2), f(a_1) and f(a_2) fail to stand in the relation f(r^2)}. Den (r^2(a_1, a_2) then reduces to the set of local denotations Den$_f$ (r^2(a_1, a_2) as f ranges over all permissible algebraic interpretations f, i.e., the set of solutions of the equation or inequality (r^2(a_1, a_2). *This set can be completely graphically represented on the plane in the usual way as an array of points corresponding to the pairs <f(a_1), f(a_2)> which stand in the relation f(r^2). But this graphic representation can also be extended to allow for the simultaneous depiction of its graphical complement, namely the depiction of the set of pairs <f(a_1), f(a_2)> which stand in the relation f(r^2). If we were to take this course, which is formally possible, it would require a graphical way of distinguishing the complementary points from the standard ones by some graphical device such as using a different color.*]

C.5. *Local denotations of Sets of Sentences.* We define the local denotation of the set S of sentences relative to an interpretation f as the set of non-empty local denotations of the sentences in S relative to f; in symbols, Den$_f$ (S).
C.5.1. Continuing the analogy with elementary algebra of the plane: The local denotation Den$_f$ (S).of a set S of equations and inequalities in “x” and “y” relative to an interpretation f is the set of non-empty local denotations of the sentences in S relative to f. Since the non-empty denotation of every sentence in S relative to f is the pair <f(“x”), f(“y”)>, the set Den$_f$ (S) is simply the singleton set {<f(“x”), f(“y”)>} That is, the pair <f(“x”), f(“y”)> is a common “solution” of all equations and inequalities in S.

C.6. *Global denotations of sets of sentences.* We define the global denotation of the set S of sentences as the set of all non-empty local denotations of members s of S relative to permissible interpretations f; that is, the set of all non-empty local denotations Den$_f$ (s) as f ranges over all permissible interpretations.
C.6.1. Continuing the analogy with elementary algebra of the plane: Let S be a set of equations and inequalities. Then the global denotation of S is the set of all non-empty local denotations Den$_f$ (s), as s ranges over S and f ranges over permissible interpretations. This is just the set of all common solutions <f(“x”), f(“y”)> of all members s of S, as f ranges over all permissible interpretations under which all members of S are true, which is commonly referred to as the “solution set” of S.

Appendix D. Semantics of “Every man loves some woman” in the Terminology of Appendix C.

D.1. Consider again the sentence, “Every man loves some woman.” In this appendix we apply the various concepts described above to this sample sentence. In particular, the local denotation of “Every man loves some woman” relative to a given interpretation f, in symbols, Den$_f$ (loves, (every man, some woman)) is the set
{<f(loves), v> // v ɇ POS$_f$ (loves, (every man, some woman))}} U

{<f(not-loves), v> // v ε NEG$_f$ (loves, (every man, some woman))}, if there is a chain function g through the sequence <f(every man), f(some woman)> such that the trace of g through <f(every man), f(some woman)> is identical with POS$_f$ (loves, (every man, some woman)) (which is the intersection: f(loves) ^ CP Every man loves some woman – i.e., the intersection f(loves) ^ f(man) X f(woman), and is the singleton empty set {Θ}, otherwise.

D.2. We note that the first half of this denotation, namely,
{<f(loves), v> // v ε POS$_f$ (loves, (every man, some woman)} =
{<f(loves), v>//v ε f(loves) ^ f(man) X f(woman)} =
{<f(loves), v>//v ε {<m1, w1>, <m1, w2> <m2, w1>} =
{f(loves), <m1, w1>, f(loves <m1, w2> , f(loves) <m2 w1>}.

D.3. And we note that the second part of this denotation, namely,
{<f(not-loves, v> // v ε NEG$_f$ (Every man loves some woman)} =
{<f(not loves), v>//v ε f(not loves) ^ f(man) X f(woman)} =
{<f(not- loves), v>//v ε {<m2, w2>}=
{f(not-loves), <m2, w2}

D.4. As a consequence we have: Den$_f$ (loves, (every man, some woman)) =
{f(loves), <m1, w1>, f(loves <m1, w2> , f(loves) <m2 w1>} U
{f(not-loves), <m2, w2}

D.5. Turning now to the relation:
f(love) = {<m1, w1>, <m1, w2>, <m2, w1>, <w2, m1>, <x,y>, <z,v> , where x,y,z,v are entities other than men or women in the domain of discourse between which the relation of loving holds, we have:
POS$_f$ (every man loves some woman)) =
f(loves) ^ f(man) X f(woman) =
{<m1,w1>, <m1,w2>, <m2,w1>}

D.6. And turning to the complement of that relation:
f(not-love) = {<m2, w2>, <x,y>, <z,v> , where x,y,z,v are entities other than men or women in the domain of discourse between which the relation of not-loving holds, we have:
NEG$_f$ (every man loves some woman)) =
f(not-loves) ^ f(man) X f(woman) =
{<m2,w2>}

D.7. Turning to the issue of the existence of a chain function g through the sequence <f(every man), f(some woman)> such that the trace of g through <f(every man), f(some woman)> is identical with POS$_f$ (loves, (every man, some woman))
[which is the intersection f(loves) ^ CP Every man loves some woman – i.e., which is the intersection f(loves) ^ f(man) X f(woman)]. Consider, the chain function g through the sequence <f(every man), f(some woman)> which assigns, to every element in the set U(f(every man)), i.e., which assigns to each of the elements m1 and m2 in U(f(every

man)), one of the subsets in f(some woman) all of whose elements x are such that <m1,x> e f(loves). There are three such subsets, namely, {w1}, {w2}, {w1,w2}, and <m1,w1>, <m1, w2> and <m2,w1>, all of which are elements of the relation f(love). Recall that this relation consists of the following pairs: {<m1, w1>, <m1, w2>, <m2, w1>, <w2, m1>, <x,y>, <z,v> , where x,y,z,v are entities other than men or women in the domain of discourse between which the relation of loving holds.[14] The trace of g through <f(every man), f(some woman)> is the set of pairs {<m1, w1>, <m1, w2>, <m2, w1>, <w2, m1>}. which is identical with POS$_f$ (loves, (every man, some woman)). (which is just the intersection: f(loves) ^ CP Every man loves some woman – i.e., the intersection f(loves) ^ f(man) X f(woman).

Appendix E. Tree Form Description of the Syntactic Structure of "Every man Loves Some Woman" and Aspects of its Semantic Interpretation

E.1. *Tree form of the sample sentence.* Thus far, we have exhibited the syntactic representation of "Every man loves some woman" in simplified linear form which does not fully exhibit the way that simple expressions are combined to form complex expressions. Their mode of combination is better exhibited by expanding expressions into a tree form which more specifically exhibits the syntactic roles of their constituent expressions, as well as their relationship to the original character string which is thus syntactically represented.

```
Character string A          Every  man      loves       some  woman
                              |     |        |  \ \       |      |
Syntactic                   UN MAN       LOVES A D   INDEF WOMAN
Representation B of            \   T         R  / /          \  T
Character string A              \|           |/ /             \|
                                 T           R1 /              T
                                  \          | /              /
                                   \         R2              /
                                    \        |             /
                                     \       |          /
                                      \      |      /
                                       \     |   /
                                            T
```

E.2. *Explanation of construction of this tree form.* Let f be an interpretation of the sentence "Every man loves some woman."

E.2.1. *Tree form of "every man."* The representational morpheme for the word "man" is MAN which is syntactically a modifier, and which is such that, when applied to the "thing label" T, which is a representational morpheme (meaning "thing"), forms the thing expression MAN
T
to which the interpretation f assigns a subset of D. The representational morpheme for the word "Every" is UN which is syntactically a modifier, and which, when applied to the thing expression MAN , forms the modifier UN MAN, which , when applied to T,
T \ T
\ |

forms the thing expression UN MAN to which the interpretation f assigns the singleton
\ T
\ |
T

set whose only member is the set f(MAN).
T

E.2.2. *Tree form of "some woman."* The representational morpheme for the word "woman" is WOMAN, which is syntactically a modifier and which is such that, when applied to the thing label T, forms the thing expression:

WOMAN ,
T

to which the interpretation f assigns a subset of the domain of discourse. The representational morpheme for the word "some" is INDEF, which is syntactically a modifier such that, when applied

to WOMAN , forms the modifier INDEF WOMAN,
T \ T
\ |

which, when applied to the thing label T, forms the thing expresssion

INDEF WOMAN
\ T
\ |
T

to which the interpretation f assigns the set of non-empty subsets of the set $f(WOMAN_T)$.

E. 2.3. *Tree form of "loves."* The representational morpheme for the word "loves" is LOVES, which is syntactically a modifier and which, when applied to the relation label R (a representational morpheme which intuitively means "relation"), forms the zero place relation morpheme $LOVES_R$,
which, when applied to the case morpheme A (which is a representational morpheme), forms the modifier

```
LOVES A
  R  /
   |/
```

which, when applied to the relation label R^1, forms the (one-place) relation expression

```
LOVES A.
  R  /
   |/
   R1
```

The case morpheme D (which is a representational morpheme) is syntactically a modifier such that, when applied to this (one-place) relation expression, forms the modifier

```
LOVES A D
  R  /  /
   |/  /
   R1 /
   | /
```

, which is such that, when applied to the relation label R^2,

forms the (two place) relation expression[37]

```
LOVES A D
  R  /  /
   |/  /
   R1 /
   | /
   R2
```

Footnote 37. The meanings of the modifiers A and D is spelled out in semantic axioms. in [22] . We can roughly indicate their intended meanings as the case morphemes A for "agent" or initiator of the action signified by the relation and "D" for the "direct object" of that action. Also, see Appendix.

Appendix F. Example of the Global Graph of The Sentence "Every Man Loves Some Woman" Relative to All Permissible Interpretations Under Which This Sentence Is True Applied to The Above Tree Form Description of the Syntactic Structure of This Sentence.

F.1. Let f be an interpretation such that:

Let f(MAN) = {m1, m2} (intuitively, the set of men in the domain of discourse),
T

Let f(WOMAN) = {w1, w2} (intuitively, the set of women in the domain of discourse).
T

Let f(UN MAN) = {{m1, m2}}, (intuitively, the singleton set of the set of men)
\ T
\ |
T

Let f(INDEF WOMAN) = {{w1}, {w2}, {w1, w2}} , (intuitively, the set of all subsets of women)
\ T
\ |
T

and

Let f(LOVES A D) = {<x1,y1>, <x2,y2>, <m1,w1>, <m1,w2>, <m2,w1> <w1,m1>, <w2,,m1>}.
R / /
| / /
R^1 /
| /
R^2

(intuitively, the set of all pairs of entities in the domain of discourse such that the first entity of the pair loves the second, irrespective of whether they both are human)

Let B be the syntactic representation of the entire character string "Every man loves some woman," as shown below:

```
B:                    UN MAN      LOVES A D   INDEF WOMAN
                        \   T        R  /  /          \   T
                          \|         |/  /              \|
                           T         R1 /                T
                            \        | /               /
                             \       R2             /
                              \      |          /
                                \    |      /
                                  \  |  /
                                     T
```

Then POS$_f$ (B)[38] = f(LOVES A D) ^ [Uf (UN MAN) x Uf (INDEF WOMAN)]

```
          R   /   /            \   T              \   T
          |  /  /                \  |                \  |
          R1  /                    T                   T
          |  /
          R2
```

= {m1,w1>, <m1,w2>, <m2,w1>,

and NEG (B) [9] = f(LOVES A D)^c ^ [Uf (UN MAN) x Uf (INDEF WOMAN)]

```
          R  /  /             \   T               \    T
          |  /  /               \  |                \  T
```

R¹ /   T   T
| /
R²

= {<m2,w2>}

Footnote 38. See Section 8, and Appendices D and E above for the meanings of Pos(B) and Neg (B)

Then, we have:

F.2. *The local denotation* $Den_f(B)$ *of B relative to f* is the set consisting of the following pairs:

<f(LOVES A D), <m1,w1>>, | /

R / /
| / /
R^1 /
| /
R^2

<f(LOVES A D), <m2,w2>>,

R / /
| / /
R^1 /
| /
R^2

and

<f(LOVES A D), <m1,w2>>,

```
R   / /
|  / /
R1 /
| /
R2
```

provided that there is a non-empty subset B1 ε f(UN MAN),

```
\   T
 \  |
    T
```

and a non-empty subset B2 ε f(INDEF WOMAN),

```
\   T
 \  |
    T
```

and a chain function g on <B1, B2>, such that the trace of g through <B1, B2> is identical with POS_f (B) (i.e., is identical with the set: {<m1,w1>, m1,w2>. <m2,w1>}. Note also that the set Den_f (B) is graphically representable as a network of nodes and connecting arcs, where the un-negated connecting arcs graphically represent the relation

```
LOVES  A D
   |    /  /
   R   / /
   |  / /
   R1 /
   | /
   R2
```

and each pair of nodes they connect graphically represents a pair of elements of the domain of discourse which stand in that relation, and where negated connecting arcs graphically represent the complement of that relation, and each pair of nodes those negated connection arc connect graphically represents a pair of elements of the domain of discourse which fail to stand in that relation.

The possible sequences <B1, B2> such that:

```
B1 ε f( UN MAN)  =    {{m1, m2}},
             \    T
                \ |
                  T
```

and

```
B2 ε f( INDEF WOMAN)  =  {{w1}, {w2}, {w1, w2}}
                     \      T
                        \  |
                           T
```

can be listed as (a) – (g), below:

(a) < {m1, m2}, {w1,w2}>

(b) < {m1, m2}, {w1}>

(c) < {m1, m2}, {w2}>

(d) < {m1, m2},{{w1},{w1,w2}}>

(e) < {{m1, m2}}, {{w2},{w1,w2}}>

(f) < {{m1,m2}},{{w1},{w2}}>

(g) < {{m1,m2}},{{w1},{w2},{w1,w2}}>

The possible chain functions through each of the sequences (a) – (g) are the following nine chain functions g_1 - g_9 :

The chain functions through (a) are:
g_1(1,m1) = {w1, w2}, g_1 (1,m2) = {w1,w2};

The chain functions through (d) and (g) are:
g_2(1,m1) = {w1, w2}, g_2 (1,m2) = {w1};

The chain functions through (c), (f) and (g) are:
g_3(1,m1) = {w1, w2}, g_3 (1,m2) = {w2};

The chain functions through (e) and (g) are:
g_5(1,m1) = {w2}, g_5 (1,m2) = {w1,w2};

The chain functions through (b), (d), (f), and (g) are:
g_6(1,m1) = {w1}, g_6 (1,m2) = {w1};

The chain functions through (c), (e), (f), and (g) are:
g_7 (1,m1) = {w2}, g_7 (1,m2) = {w2};

The chain functions through (f), and (g) are:
g_8 (1,m1) = {w1}, g_8 (1,m2) = {w2};

The chain functions through (f) and (g) are:
g_9 (1,m1) = {w2}, g_9 (1,m2) = {w1}.

For each of the sequences (a) – (g), and for each of the possible chain functions g_1 - g_9, the traces t_1 - t_9 of g_1 - g_9 through the sequences (a) – (g) are as follows:

The trace t_1 of g_1 through (a) is: {(m1,w1), (m1,w2), (m2,w2)};

The trace t_2 of g_2 through (d) and (g) is: {(m1, w1), (m1,w2), (m2,w1)};

The trace t_3 of g_3 through (c), (f), and (g) is: {(m1w1), (m1,w2), (m2,w2)};

The trace t_4 of g_4 through (d) is: {(m1,w1), (m2,w1), (m2,w2)};

The trace t_5 of g_5 through (e) and (g) is: {(m1w2), (m2,w1), (m2,w2)};

The trace t_6 of g_6 through (b), (d), (f), and (g) is: {(m1w1), (m2,w1)};

The trace t_7 of g_7 through (c), (e), (f), and (g) is: {(m1w2), (m2,w2)};

The trace t_8 of g_8 through (f), and (g) is: {(m1w1), (m2,w2)};

The trace t_9 of g_9 through (f), and (g) is: {(m2,w2), (m2,w1)}.

Note that the trace t_2 of g_2 through each of the sequences (d) and (g) is the only trace among t_1 - t_9 that is identical with the positive relational profile of B relative to f.:

```
POSf (B) = f(  LOVES A D ) ^  [Uf (UN MAN)  x  Uf (INDEF WOMAN)]
                  R   /   /                  \   T                    \   T
                  |  /  /                     \  |                     \  |
                  R1  /                          T                        T
                  | /
                  R2
```

which is the set: {m1,w1>, <m1,w2>, <m2,w1>}. We can readily associate a unique local graph with this trace, namely, the graph which is a diagrammatic depiction of the local denotation of the sentence, “Every man loves some woman” relative to the interpretation f (as shown).

We diagrammatically depict the global denotation of this sentence relative to all permissible interpretations f of the relation

```
LOVES  A D
    |    /  /
    R   / /
    | / /
    R1 /
     |/
    R2
```

restricted to the Cartesian product of the set of men (i.e., {m1, m2}) and the set of women, (i.e., {w1, w2}), as the linked array of the nine local graphs associated with the . traces t_1 - t_9, namely the linked array of the nine local graphs: (1), (2), (3), (4), (5), (7), (8), (9), (11) of Figure 3.

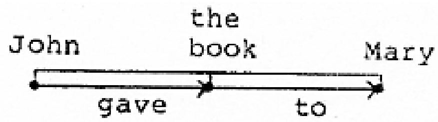
Appendix G. Local Graphs of NL Sentences, Generally Considered.

G1. *Components of Local Graphs.*

G.1.1. *Nodes, Arcs, and Paths.* Local graphs are composed of two types of basic graphical elements: nodes and arcs. Nodes represent elements of the underlying domain of discourse and arcs represent relations on those elements.
G.1.2. *Simple and Compound Arcs. Simple arcs* are arcs that join at most two entities. There are three types of simple arcs: (i) arrows, which joins at most two nodes and which can be barred or unbarred; an unbarred arrow represents a relation whose relata are elements of the underlying domain of discourse, and considered in the order indicated by the direction of the arrow, and a barred arrow represents the complement of that relation; (ii) dotted lines, which represent the identity relation when unbarred, and the non-identity relation when barred, and which join either two points to represent that they represent the same or different elements of the underlying domain of discourse, or two arrows to represent that they represent the same relation if both are barred or both are unbarred, or to represent complementary relations if one arrow is unbarred and the other is barred; and (iii) dashes, which represent the logical conjunction of the entities represented by the graphical entities it joins. *Compound Arcs* are arcs formed by joining two or more simple arcs with a graphical unit called a "brace," and represent many-place relations composed of those simple arcs. An arc that is not a constituent of a compound arc is said to be major.
G1.3. *Dot Paths, Arrow Paths, and Mixed Paths.* A path is a simple or compound arc taken together with nodes it joins. If the constituent arcs of the path are all dotted lines, the path is called a *dot path*. If the constituent arcs of the path are all arrows, the path is called an *arrow path*, if the constituent arcs of the path are both dotted lines and arrows; the path is called a *mixed path*. An arc which is a constituent of a path is said to be a major constituent of that path if it is not itself a constituent of another constituent of that path. A path is said to be *barred or unbarred* according as its major constituent is barred or unbarred. A path represents that the elements of the domain of discourse respectively represented by the nodes of the path stand in the relation represented by the path. Arrow paths and mixed paths represent lexical relations, the place number of which corresponds to the number of nodes in the path. A single node placed at the origin or terminus of an arrow signifies respectively that the element represented by the node is in the domain or range of the relation represented by the arrow.

Appendix H. Examples of Dot Paths, Arrow Paths, and Mixed Paths. An unbarred braced arrow path joining two points represents that the elements of the universe of discourse represented by those points stand in the relation represented by the barred or unbarred arrow. An unbarred or barred dotted line joining two points represents that the elements of the universe of discourse represented by those points are or are not identical. Whether a given single arrow represents a two place or one place relation is determined by whether exactly two points or exactly one point occurs with the arrow. Placement of a point at the origin or terminus of an arrow signifies respectively that the element represented by the point is in the domain or range of the relation represented by the arrow. An analogous situation holds for relations of higher place number. We note that a dotted line can represent only the identity relation, which is binary, and so must always join two points.

Examples of line, arrow, and mixed paths are given in Figure 6 below:



(John gave the book to Mary)

(John did not give the book to Mary)

(John gave the book)

(The book was given to Mary)

John did not give the book)

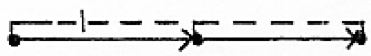
The book was not given to Mary)

(John gave the book and the book was given to Mary)

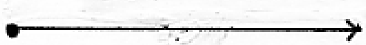
(John did not give the book and the book was not given to Mary)

(John gave the book and the book was not given to Mary)

(John did not give the book and the book was given to Mary)

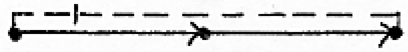


(It is false that John gave the book and the book was given to Mary)



(John gave to Mary)

(John did not give to Mary)



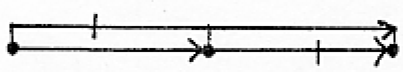
(It is false that John gave the book and the book was given to Mary)

(It is false that John did not give the book and the book was given to Mary)

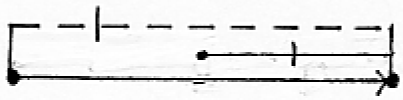
(It is false that John did not give the book and the book was not given to Mary)

(John did not give the book but gave to Mary)

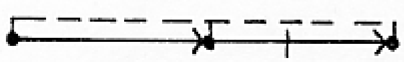
(John gave the book but not to Mary)



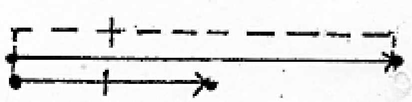
(It is false that John gave the book but not to Mary)



(It is false that John gave to Mary but the book was not given to Mary)



(John gave the book but the book was not given to Mary)



(It is false that John did not give the book but gave to Mary)

Figure 6

Comment regarding Figure 6. Note that there are two types of braces used here, one type drawn as a solid line indicating that the arrow paths it joins depict sub-relations of a given relation, and one type drawn as a dashed line indicating that the arrow paths it joins depict relations which are conjncts of the given relation. A second use of dashed line here is to allow the graphical expression of the complement of the relation it spans..

Appendix I. Examples of Graph based Deductive Entailment Using the Weak Form of the Positive Deductive Graphical Paradigm, and the Negative Deductive Graphical Paradigm.

Recall *the weak form of the positive deductive paradigm*: *The sentences P1, ..., Pn deductively entail the sentence C if the graph of C is contained in the graph of the join of P1, ..., Pn.*

Recall also *the negative deductive paradigm*: *The sentences P1, ..., Pn deductively entail the sentence C if and only if the graph of not-C (the negation of C) is graphically incompatible with the graphical combination of the graphs of P1, ..., Pn.*[11]

I.1. General comments on the following diagrams: (i) local graphs are shown as individual diagrams, and global graphs are shown as linked connections among local graphs, the linkages depicted by dotted lines which have the usual meaning of the identity relation; (ii) sample dotted lines are drawn between nodes and between arcs which depict identical entities among local graphs; (iii) dotted lines connecting sample local graphs are used to avoid cluttering these diagrams; (iv) also – to avoid clutter - only unnegated paths are drawn in, so that, the absence of a given unnegated path between two or more nodes represents that a negated path is to be understood as connecting those nodes. (v) deductive relations among global graphs are indicated by bold arrows with origins situated at the entailing global graphs and their termini situated at their entailed global graphs; (vi) where the number of local graphs in a given global graph is too large to fit into these one-page diagrams, we express their continuation by the expression "etc."; (vii) the English sentences which given global graphs represent are typed in their informal – i.e., pre-formalized form - above the global graphs which represent them; (viii) the global graph representing the negation of a given sentence is the complement of the global graph representing that sentence; (ix) recall that a given global graph is a linked array of local graphs, each local graph depicting a local denotation of the represented sentence relative to a given denotation of that sentence, the definitions of which are summarized later in Appendix I which follows these examples. (x) in machine applications to sentences with very large numbers of local graphs in global graphs of those sentences, it may be expedient to use random samples of those local graphs to yield approximate determinations of their deductive connections.

I.2. Comments on the diagrams regarding the application of each of the two paradigms: : Recall: The weak form of the positive deductive paradigm states that sentences

P1, ..., Pn deductively entail the sentence C *if* the graph of C is *graphically included* in the graphical join P1^ ... ^Pn of the graphs of P1, ..., Pn, and that negative deductive paradigms states that sentences P1, ..., Pn deductively entail the sentence C *if and only if* the graph of not-C is *graphically incompatible* with every local graph in the graphical join P1^ ... ^Pn of the graphs of P1, ..., Pn.

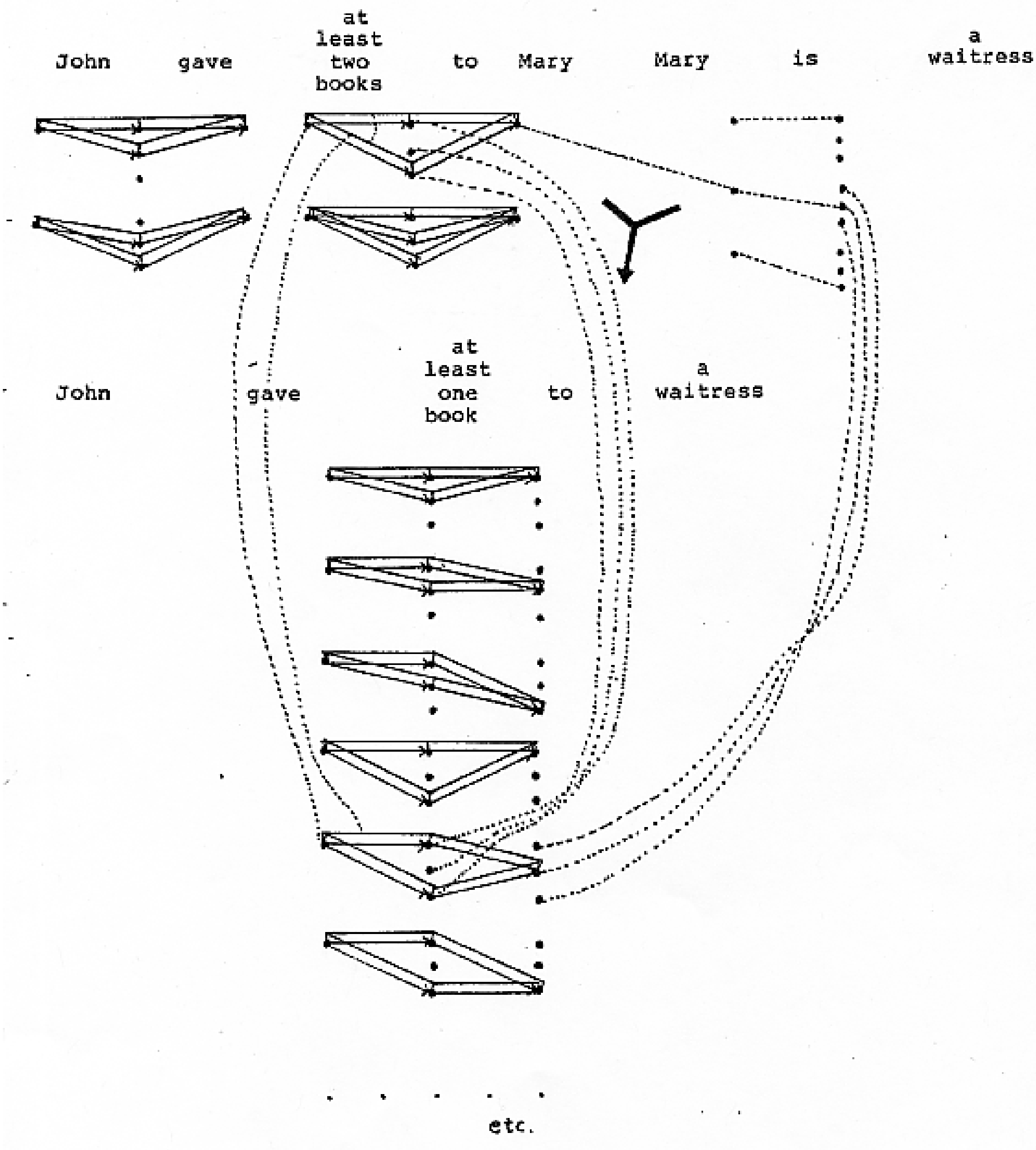


Figure 7

Comment regarding Figure 7. (Note first that the reading we make for the article "a" in this example is in the sense of "some," so that "a waitress" has the meaning of "some waitress.") Letting (A) be the sentence, "John gave at least two books to Mary," letting (B) be the sentence "Mary is a waitress," letting (C) be the sentence "John gave at least one book to a waitress. letting ~(C) be the negation of (C), namely "John gave no books to any waitress," we have, following the dotted lines from any local graph of the join of the graphs of (A) and (\B) to any local graph of (C), that the latter local graph is included in the former, which implies, by *the weak deductive graphical paradigm*, that (A) and (B) together entail (C). Under the same assumptions, and noting that the graph of ~(C) is a graph similar to the graphs of (C) but all of whose braced local graphs are negated and hence incompatible with every local graph of the join of the graphs of (A) and (B), the machine concludes, by *the negative deductive graphical paradigm*, that (A) and (B) together entail (C).

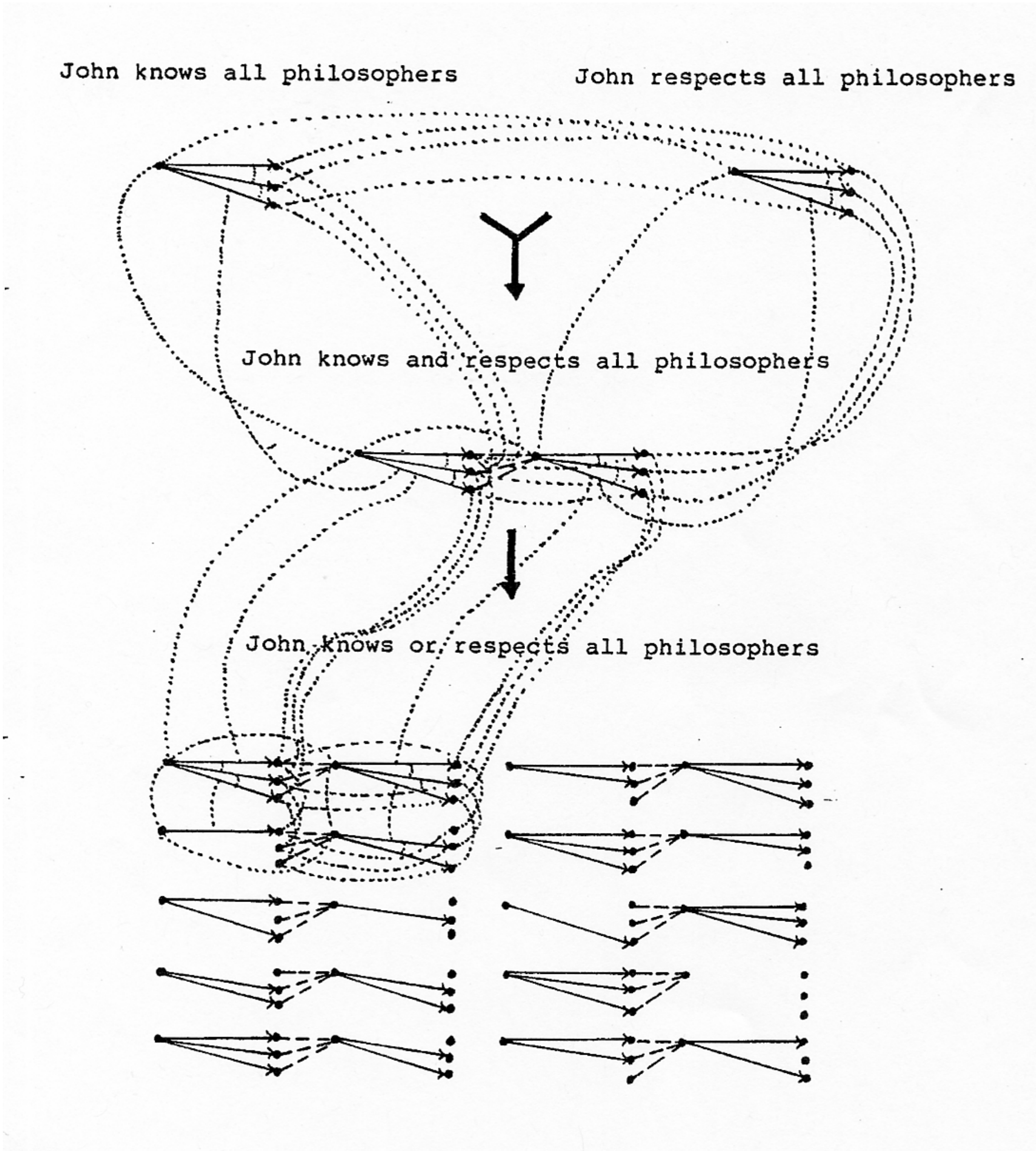


Figure 8

Comment: on Figure 8: we see the use of dashed lines (- - -) to depict the conjunction of the local graphs they connect.

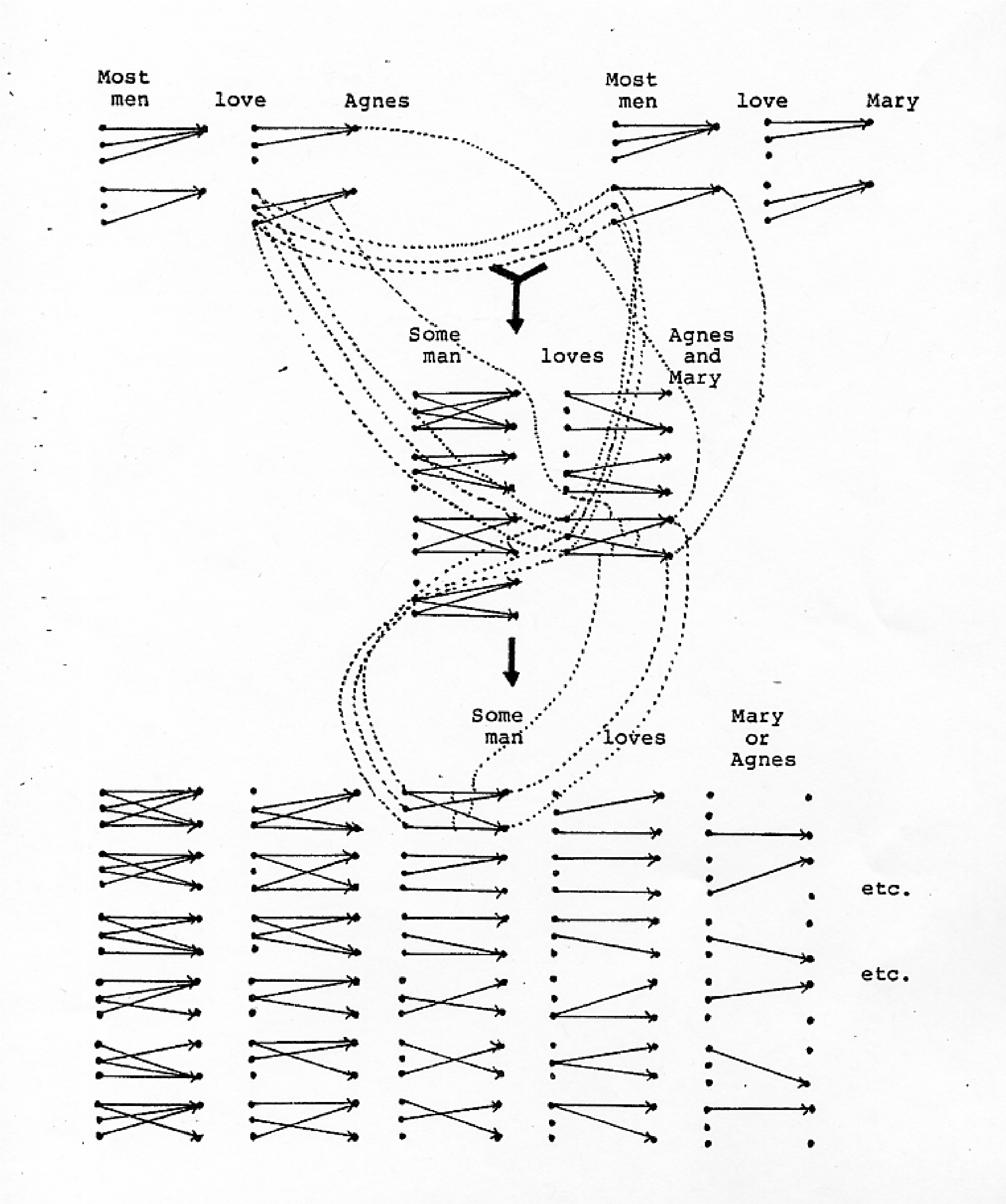

Most men
love
Agnes
Most men
love
Mary
Some man
loves
Agnes and Mary
Some man
loves
Mary or Agnes
etc.
etc.

Figure 9

Comment: regarding Figure 9: Letting (A) be the sentence, “Most men love Agnes,” letting (B) be the “Most men love Mary,” letting (C) be the sentence “Some man loves Agnes and Mary,” letting ~(C) be the negation of (C), namely “No man loves both Agnes and Mary,” we have, following the dotted lines from any local graph of the join of the graphs of (A) and (B) to any local graphs of (C), that the latter is included in the former, which implies, by *the weak deductive graphical paradigm*, that (A) and (B) together entail (C). Under the same assumptions, we note that the local graphs of ~(C) are similar to the graphs of (C) but are such that no pair of arrows issuing from the same “man node” in any these are such that one of these arrows terminates at an “Agnes node” and the other of which terminates at a “Mary node.” which is to say that every local local graphs of ~(C) is incompatible with every local graph of the join of the graphs of (A) and (B); hence, by *the negative deductive graphical paradigm*, the machine concludes (again) that (A) and (B) together entail (C).

Under the same assumptions regarding (A), (B), and (C), and letting (D) now be the sentence, “Some man loves Mary or Agnes, and letting ~(D) be the negation of (D), namely “No man loves either Mary or Agnes,” we have the following, following the dotted lines from any local graph of (C), that is from any local graph of “Some man loves Agnes and Mary,” to (D), that is, to “Some man loves Mary or Agnes,” we note that every local graph of (D) is included in some local graph of (C), which implies, by *the weak deductive graphical paradigm*, that the sentence (C) entails the sentence (D). Under the same assumptions, and noting that there is only one possible local graph of ~(D), namely the local graph in which there are no unnegated arrows, it follows then, that every local graph of ~(D) is incompatible with every local graph of (C) (inasmuch as the local graphs of ~(D) are similar to those of (C) and the fact that every local graph of (C) has at least one unnegated arrow). Thus by *the negative deductive graphical paradigm*, the machine concludes that (C) entails (D).

By similar considerations, a human (or a machine) can arrive at the conclusion that (A) and (B) together entail (C). Alternatively, we (or a machine) can arrive at the same conclusion by virtue of the transitivity of entailment.

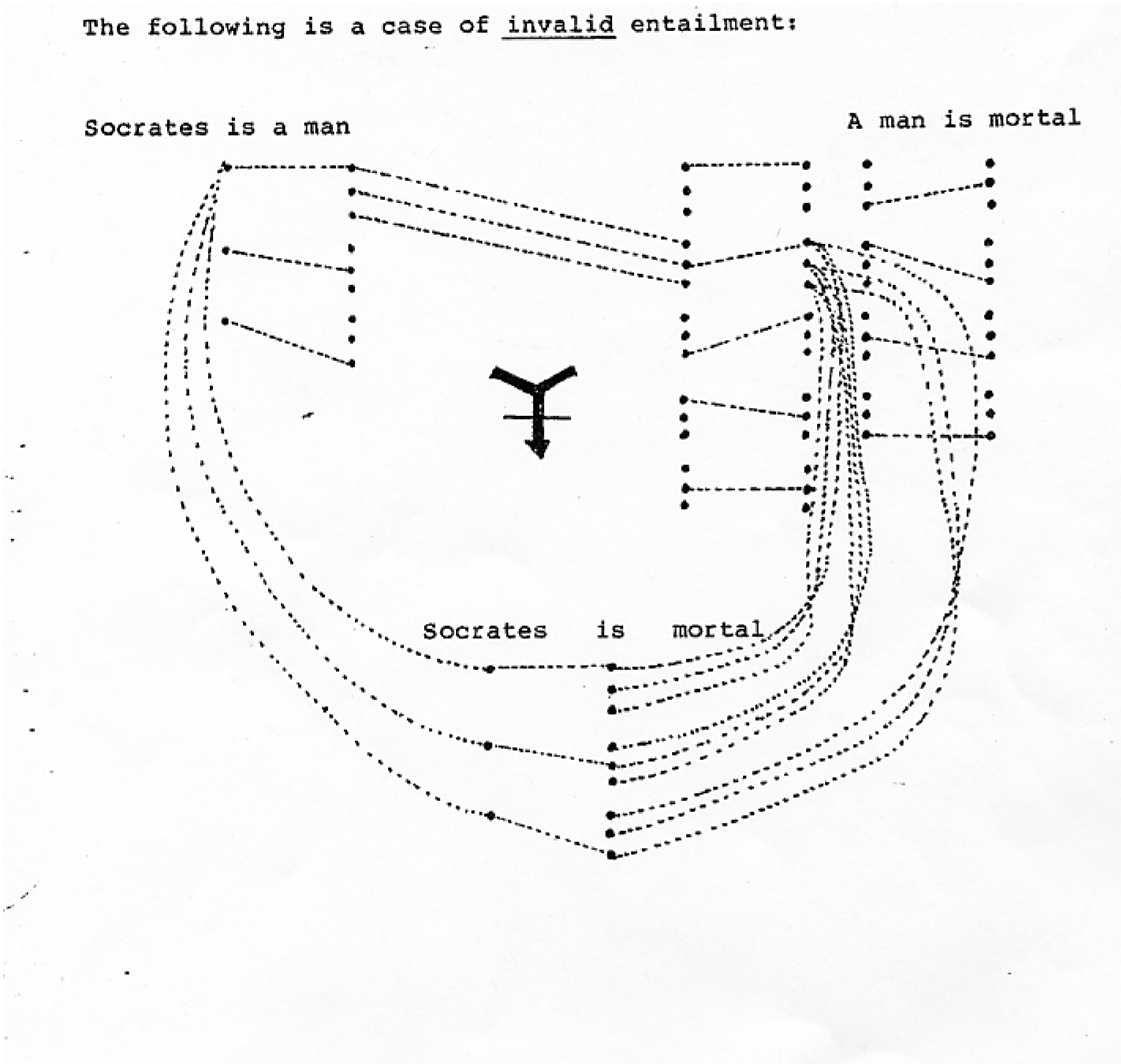


Figure 10

Comment regarding Figure 10: Letting (A) be the sentence, “Socrates is a man,” letting (B) be the sentence “A man is mortal,” letting (C) be the sentence “Socrates is mortal,” letting ~(C) be the negation of (C), namely “Socrates is not mortal,” we note that, following the dotted lines from the local graphs of the join of the graphs of (A) and (B) to the local graphs of (C), that it is not true that every local graph of the join of the graphs of (A) and (B) includes some local graph of C) (for example, the local graph shown of the join of the graphs of (A) and (B) fails to include the (shown) topmost local graph of (C), so *the weak positive deductive paradigm fails to yield* that (A) and (B) together entail (C). On the other hand, noting that the graph of ~(C) is a graph similar to the graphs of the join of the graphs of (A) and (B) but which has no dotted lines connecting the “Socrates” node with any of the “mortal” nodes, which renders every such graph incompatible with every graph of that join (inasmuch as every graph in this join does have dotted lines connecting the “Socrates” node with the “mortal” nodes), enabling a machine to conclude that *the* *sentences (A) and (B) together do not deductively entail the sentence (C).* Thus we have a case here where the weak positive graphical paradigm does not imply non-entailment, but the negative deductive paradigm does imply non-entailment. Put differently, the weak positive deductive paradigm provides a sufficient but not necessary condition of deductive entailment whereas the negative deductive paradigm provides both a necessary and sufficient condition for deductive entailment

We compare this case with the following familiar instance of valid entailment, and whose validity can be established by both the weak deductive positive paradigm and the negative deductive paradigm:

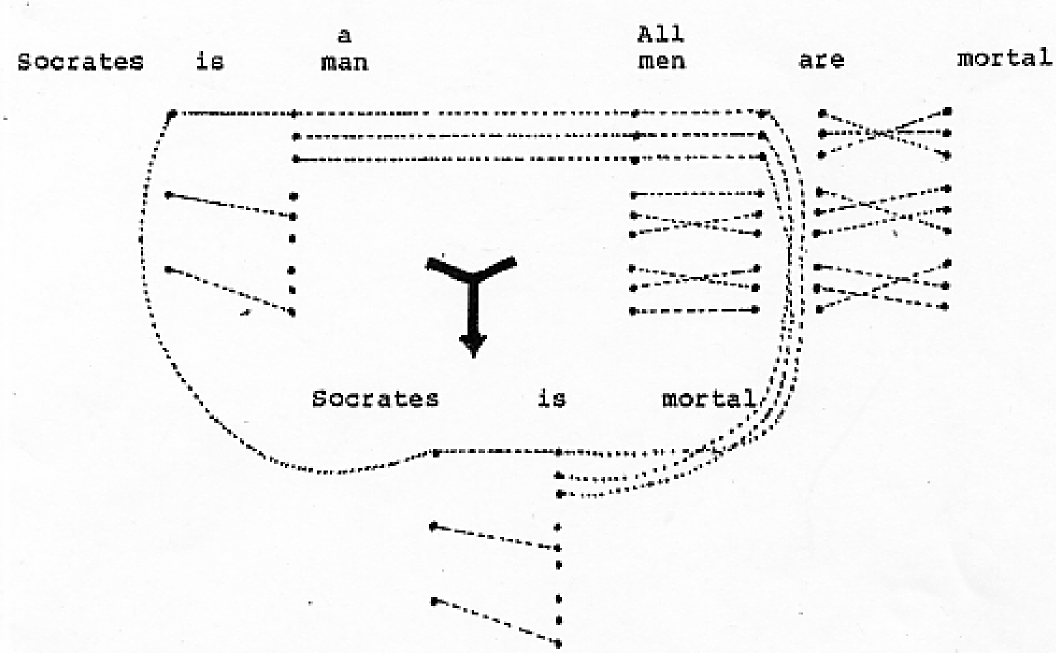


Figure 11

The following Figure 12 is an example of valid entailment of a conclusion from *three* premises:

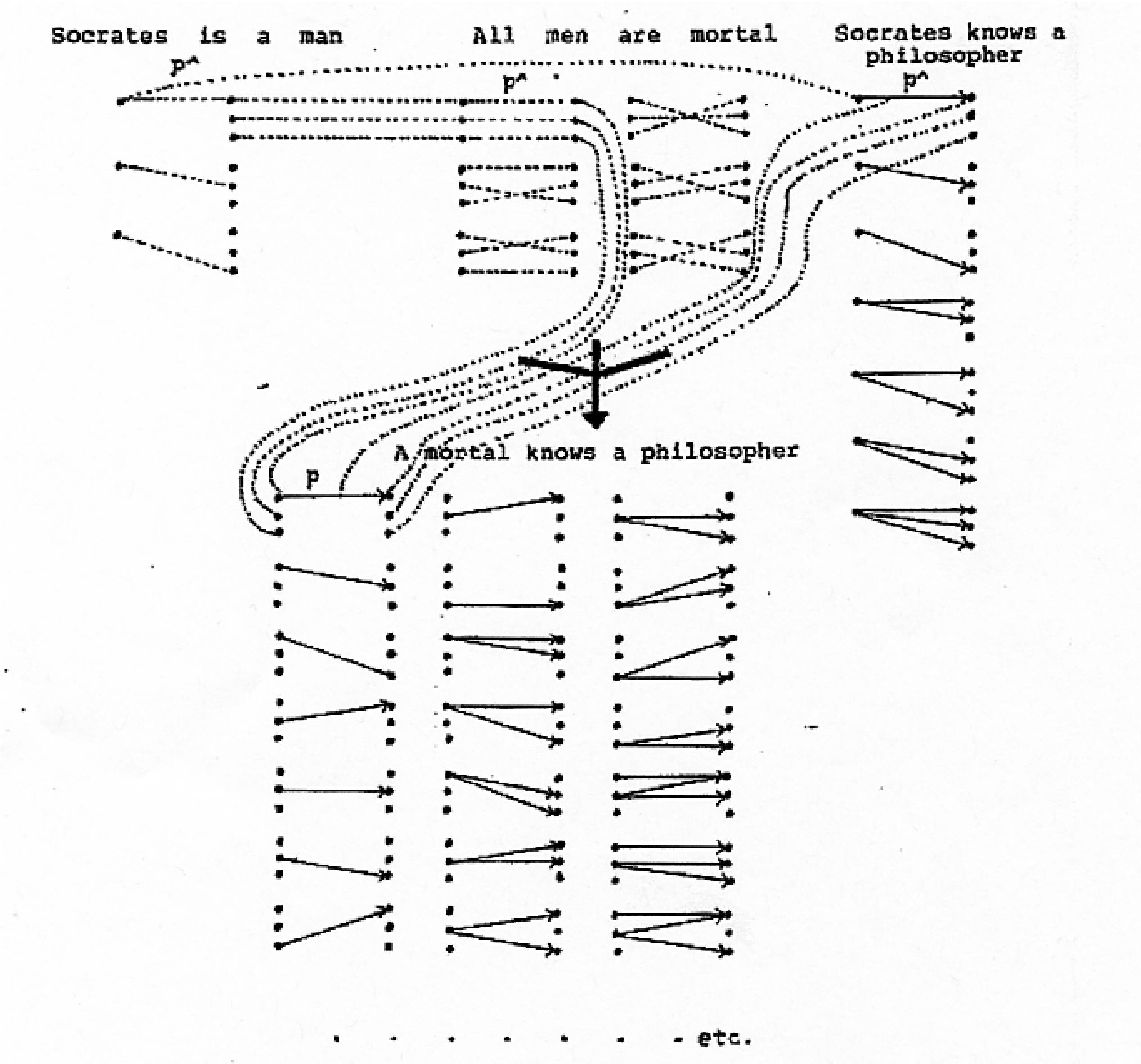


Figure 12

Comment. (As in comments on Figures 8, 10, and 11, we read "a" in the sense of "some.") This example illustrates a case involving three premise sentences as shown, and which can be established either by the weak positive deductive graphical paradigm or the negative deductive paradigm..

Appendix J. More on the Application of the Negative Deductive Graphical Paradigm to the Graphs of Appendix I.

J.1. Regarding the application (illustrated on page 58) of the weak positive deductive paradigm to show the deduction of (C) “Socrates is mortal” from the sentences, (A) “Socrates is a man,” and (B) “All men are mortal.” We can also see how the negative deductive paradigm can be applied to show this same deduction, by noting that every local graph of the join of the global graphs of sentences (A) and (B) is, *incompatible with the only local graph of the negation ~(C) of (C), and in which no dotted lines occur.* We recall how local graphs are to be read: For any local graph g and for any sequence S of point banks in g, the absence of a path and joining successive nodes in S is to be regarded as *standing for a barred path* joining the nodes in S which is similar to the other paths in S.

J.2. Regarding the application (illustrated on page 54) of the weak positive deductive paradigm to show the deduction of (C) “John knows and respects all philosophers” from the sentences, (A) “John knows all philosophers” and (B) “John respects all philosophers.” and the deduction of (D) “John know or respects philosophers.” from (C). We can now also see how the negative deductive paradigm can also be applied to show each of these two deductions: First, we note that every local graph of the join of global graphs of sentences (A) and (B) is *incompatible with every local graph of the negation ~(C) of (C), each of which is a local graph composed of two dash-joined local graphs at least one of which contains at least one barred arrow, hence – by the negative deductive paradigm – (C) is an deduction of (A) and (B. Second,* we note that the single local graph of the dash-join of the global graphs of sentence (C) is *incompatible with every local graph of the negation ~(D) of (D), each of which is composed of the dash-join of two local graphs, at least on of its components contains at least one barred arrow, hence is incompatible with the only local graph of (C) (in which no barred arrows occur).*Analogous considerations would apply to establish the deduction of (D) from (A) and (B), directly. The reader can verify that the application of the negative deductive paradigm would yield each of the deductions established by applying the positive weak deductive paradigm discussed on pages 42,44, 45, and 46.

Appendix K. General Comments on Proposed Syntactic Structure

K1. *No consensus regarding “proper” syntactic structure.* There is no consensus regarding the “proper” syntactic structure of natural language character strings, and different structures appear useful for different purposes. While there are ordinarily many possible syntactic structures that can be assigned to a given character string, the syntactic function we propose in this paper is one to which we refer as meeting a certain *homology requirement.*

K.2. *The homology requirement*. Syntactic representations of character strings which meet the homology requirement are representations whose structure mirrors – as closely as possible - the intuitive ways that those character strings are understood. In other words, the part-whole relationships in syntactic representations of a natural language sentences should mirror the intuitive part-whole relationships in those sentences, which, in part, implies that sequential orders are preserved. The point of the homology requirement is to enable syntactic structures of natural language character strings to bear *simple direct relationships to the sequential structure of characters comprising those strings* in order to expedite their real-time assignment.

K3. *Hypothesis underlying the homology requirement.* We hypothesize that syntactic representations of natural language character strings which meet the homology requirement are probably closer to how humans intuitively structure those character strings than syntactic structures that do not meet it.
That is, when a language user understands a given natural language character string, we presume that he or she arrives at that understanding by identifying its meaning bearing substrings and their mode of their organization. As a machine analogue of this presumed way in which humans arrive at understandings of natural language character strings, it is useful to have machines impose syntactic representations of those character strings which identify and recursively organize the meaning bearing substrings of those character strings in a manner which *minimally alters the order in which they occur in those character strings.*

K.4. *Representational morphemes.* The syntactic representation of a natural language character string forms a pattern of interconnections of its meaning-bearing parts which is recursively built up out of minimal meaning bearing parts called *representational morphemes*. These are explicit symbolic realizations of explicit or implicit morphemes (in the usual linguistic sense)..

K.5. *Syntactic assignment to natural language character strings*. A syntactic assignment is a function which, in application to a natural language character string X, assigns a representational morpheme to each occurrence of a minimal meaning bearing substring of X, thereby converting X into a string X^ of representational morphemes, and assigns a label to every substring of X^ corresponding to the meaning bearing substring of X which it converts, and which indicates its grammatical function in X^ and, derivatively, the grammatical function of the meaning bearing character string in X to which it corresponds. The syntactic structure of a character string which is understood as a sentence can be schematically indicated in the form $r^n (a_1, ..., a_n)_{p,s,c}$, where r^n is an n-place relation expression composed of a base relation $(r^n)^B$ attached to which are zero or more modifiers, together with an *ordered set of zero or more case markers* $b_1, ..., b_m$ indicating the semantic roles to be played by each of the n thing expressions $a_1, ..., a_n$ occupying the n places in $r^n(a_1, ..., a_n)_{p,s,c}$, and p, s, and c are three orderings on the n thing expressions $a_1, ..., a_n$, referred to respectively as *the relative place ordering, the relative scope ordering, and the relative case ordering on $a_1, ..., a_n$*. The *relative place ordering p on $a_1, ..., a_n$* determines the order in which $a_1, ..., a_n$ are to be taken relative to the n-place relation denoted by r^n , in the sense that the thing expression a_i is to occupy the p(i)th argument place of that relation. The *relative scope ordering s on $a_1, ..., a_n$* determines the scopes of each of the modifiers attached to r^n which govern each of $a_1, \ldots, a_n$, and *the relative case ordering c on $a_1, ..., a_n$* determines the association of the case markers $b_1, ..., b_m$ with the thing expressions $a_1, ..., a_n$. , in the sense that, for each i, $1 \leq i \leq n$, $c(a_i)$ is the order of that case among $b_1, ..., b_m$ which applies to the thing expression a_i. The truth condition of the denotation of $r^n (a_1, ..., a_n)_{p,s,c}$ under an interpretation is regarded as describing an "event" or "state of affairs" to the effect that the denotations of the n thing expressions $a_1, \ldots, a_n$ stand in the relation denoted by r^n relative to the three orderings p, s, and c. For most sentences of English, case expressions are usually placed adjacent to the thing expressions they govern, and both relative place and scope orderings are usually the "identity orderings," that is, they coincide with the order of occurrence of the thing expressions they govern. But this is not the situation for all sentences of English, nor for sentences of many other languages. The syntactic structure of sentences must take into account each of these special orderings. For example, different relative place orderings p correspond to the difference between "Every man loves some woman" and "Some woman loves every man," different relative scope orderings s correspond to the difference between "Every man loves some woman" and "Some woman is such that every man loves her," and different relative case orderings c correspond to the difference between "Every man loves some woman" and "Every man is loved by some woman."

Appendix L. General Comments Regarding the Estimation of Range and Limitations of Proposed Approach

L.1. Regarding *range* of application: there appears to be no essential limit to the kinds of grammatical structures to which the approach described here can be applied. In particular, in [22] we describe its application to deductive relations turning on the semantic structures of the following grammatical construction: determiners of arbitrary sorts including but not restricted to ordinary quantifiers, sentential connective, phrasal connectives, modal and temporal operators, branching quantifiers, pronominal and proverbal referential constructions within and across sentence boundaries, modifier constructions of arbitrary sorts, including phrasal, clausal, adjectival, and adverbial constructions.

L.2. It is not clear regarding the degree to which the extensional semantics in the approach described here can handle natural language deductions which appear to essentially involve intensional semantic structures. In [ ] we suggest ways to to "extensionalize" a variety of semantic structures which are usually treated as intensional in the literature.

L.3. We claim in this paper that the proposed graphical structures for NL sentences are designed to facilitate near-instantaneous determinations of whether given sentences are deductive consequence of given sets of sentences of arbitrary finite size.

L.4. The syntactic theory proposed in this paper attempts to assign to NL sentences syntactic representations whose internal composition parallels the intuitive way a language user might intuitively form them.

L.5. *Circumventing the formulation of tree forms for arbitrary NL sentences*: One need not formulate tree forms of English sentences in applications in order to display their global graphs. One can proceed directly to constructing their local and global graphs, roughly as follows:

(i) Identify *the major thing and relation expressions* in this sentence, where the major thing expressions in this sentence are those thing expressions in this sentence which are not embedded in other expressions, and where the major relation expression in this sentence is the n-place relation expression which relates the n major thing expressions in this sentence.

(ii) Identify the *base thing expressions* in this sentence, which result by stripping all modifiers (hence determiners) from the major thing expressions in this sentence. The base thing expressions in this sentence are "men" and "woman," and the thing expressions containing them are "All men" and "some woman."

(iii) Identify the *major relation expression* in this sentence and the number of its argument places. The major relation expression in this sentence is "loves" and the number of its argument places is 2.

(iv) For every interpretation f under which the sentence, "All men love some woman," is true, identify the *denotations of its base thing expressions* under f. Generally, for every interpretation f, and for every base thing expression in a given sentence, decide how many entities in the domain of discourse would be members of the denotation of each under an interpretation.

(v) Identify the *modifiers on the base thing expressions* which form the major thing expressions in which they occur. These modifiers would include their determiners, which are important inasmuch as they determine that subset of its power set which will be graphically represented;

(vi) Form the *local graph* of the sentence in question relative to the interpretation f as a graphical sequence[40] the nodes in a vertical array[40] – each of which collectively represents the set of entities in its denotation. For each n-tuple in the denotation of the major relation of the sentenec, draw an *unbarred arrow* joining the n nodes representing an n-tuple of entities among which the relation holds, and a barred arrow joining the n nodes representing an n-tuple of entities among which the relation fails to hold. This array is the local graph of the sentence.[41, 42]

(vii) Form the *global graph* of the sentence in question relative to all permissible interpretations as the linked array of all local graphs of this sentence determined by permissible interpretations.

L.6. Regarding the circumventing of tree forms in our sample sentence, “All men love some woman.”

(i’) The *major thing expressions* in this sentence are “All men” and “some woman.” The *relation expression* in this sentence is “loves.”

(ii’) The *base thing expressions* in this sentence, which result by stripping all modifiers (hence determiners) from the major thing expressions in this sentence, are “men” and “woman.”

(iii’) The *major relation expression* in this sentence is “loves,” and and the number of its argument places is 2.

(iv’) Let f be an interpretation under which this sentence is true. Then the *denotations of its base thing expressions* under f, under the assumption that the denotation f(“man”) of the base thing expression “man” under f consists of two entities, m1 and m2, and under the assumption that the denotation f(”woman)” of the base thing expression “woman” under f consists of two entities, w1 and w2.

(v’) The *modifiers on the base thing expressions* in this sentence are their determiners, which “All” and “some,” which determine the non-empty subsets of the thing expression under f, which they modify, and which are to be graphically represented.
More particularly, the determiner “all,” applied to the base thing expression “men,” determines the singleton set whose only element is f(“men”). i.e., the denotation of “men” under f. And the determiner “some,” applied to the base thing expression “woman,” determines the set of all non-empty subsets of f(“woman”), i.e., the denotation “woman” under f. Generally, the determiner “some,” applied to a set, forms the set of all non-empty subsets of that set; accordingly, f(“all men”) is the singleton set whose only member is the set f(“men”) i.e., the set of all men, and f(“some woman”) is the set of all non-empty subsets of the set f(“woman”), i.e., the set of all subsets of the set of women.

(vi’) Form the *local graph* of the sentence, “All men love some woman” relative to the interpretation f as a two-term graphical sequence[40] whose first term consists of the nodes (there are only two) – in a vertical array[40] – which collectively represent the set f(“men”), and whose second term consists of the nodes (there are only two) – in a vertical array[40] – which collectively represent the set f(“woman”), and which are such that, for each pair <a,b>in the relation f(“loves”), draw an *unbarred arrow* joining the node representing a to the node representing b, for each pair <a,b>in the relation $[f(\text{“loves”})]^c$, i.e., in the relative complement of the relation f(“loves”), draw a *barred arrow* joining the node representing a to the node representing b. This is the local graph of the sentence, “All men love some woman” determined by the interpretation f.[41, 42]

(vii’) Form the *global graph* of the sentence, “All men love some woman” relative to all permissible interpretations as the linked array of all local graphs of this sentence determined by permissible interpretations. With respect to Figure 3, this is the linked array of local graphs (1), (2), (3), (4), (5), (7), (8), (9), (11).

Footnote 41. Noting that the only way two interpretations of the sentence “All men love some woman” can differ is in what they assign to the relation expression “loves,” we have that as f ranges over all interpretations which assign a relation f(“loves”) to the relation expression “loves” included in the Cartesian product f(“men”) x f(“women”) which is such that, for all entities a in the domain f(“men”) there is a set z in the domain of f(“woman”) such that, for all b e z, the pair <a,b> e f(“loves”) if and only if there is an unbarred arrow joining the nodes representing a and b, and the pair <a,b> e $[f(\text{"loves"})]^c$ if and only if there is a barred arrow joining the nodes representing a and b.

Footnote 42. Alternatively, one could develop the global graph of this sentence as the graphical depiction of the global denotation of this sentence, that is by identifying the traces of the chain functions through the sets f(“all men”) and f(“some women”), as f ranges over all permissible interpretations f such that f(“man”) and f(“woman”) each contain two entities. See Section 10 regarding this alternative. However, it is easier and more intuitive to identify all local graphs of a sentence by considering only the connection between interpretations and the graphs they induce.

Appendix M. Range of Applicability

M1. *Regarding size of text base.* The ideas and general approach to machine deduction indicated above, while restricted in this short paper to a detailed application to the simple sentence, “All men love some woman,” have a considerable range of applicability which will be illustrated in forthcoming papers.[42] The main idea of this paper (and of its generalizations) is that, by using graphical representations such as those described for this sample sentence, deductive entailments among very large numbers of NL sentences can be simultaneously and near-instantaneously identified. In particular, given a graphical representation of a very large text base and the graphical representation of a given sentence, it can be near-instantaneously determined whether the given sentence is deductively entailed by that text base by matching the graphical representation of the given sentence against the graphical representation of that text base.

Footnote 42. The material in these papers will largely be restatements of parts of [22].

M2. *Regarding range of sentences.* In particular, we will describe the applicability of our approach to machine determination of deductive relationships among sentences incorporating a variety of grammatical structures embodying a wide range of phrasal and clausal modifier constructions, and pronominal and proverbal referential constructions within and across sentence boundaries.

M.3. *Grounding in model theoretic semantics.* Our graphic representations have a surface similarity to certain types[43] of semantic networks, but differ significantly from them inasmuch as they are *explicitly grounded* in a version of model theoretic semantics which mediates between syntactic structures for NL sentences and graphical structures which depict their meanings.

Footnote 43. I am thinking here of the network contructions of Sowa,

M.4. *Results are approximative.* What we have described in this paper is a theoretical model for near instantaneous machine determination of deductive connections among NL sentences. In practice, results may be only approximative, which can occur in any of several ways:
(i) The underlying reading of a character string which determines its syntactic structure (such as treating “a” as “some” and “woman” as not necessarily singular in our sample sentence) may not be the most appropriate reading of that character string in the context in which that character string occurs, and may lead to erroneous determinations of its deductive connections;
(ii) The size of denotations of base thing expressions may be too small and their local graphs consequently too restrictive, to yield an accurate determination of deductive connections. (In our sample sentence, we had only two entities in the denotations of “men” and of “woman”). Generally, it may be best to assign base thing expressions with small denotations, and retain them if the deductive determinations they induce are intuitively correct;
(iii) Even if the size of denotations of base thing expressions is small and does not itself lead to erroneous determinations of deductive connections, the global graphs of individual sentences and/or the global graphs of their joins may (because of their syntactic structures) be very large, and need to be curtailed in some way, possibly by randomly choosing a subset of local graphs among them, either in the process of their generation, that is, before all their component local graphs are generated, or by imposing some arbitrary restriction on them. We do not consider different sampling strategies or other ways of curtailing the sizes of global graphs in this paper, but different ways of curtailing the size of global graphs can be “tested” to determine which among them yields deductive determinations among given sentences which are the most intuitive.

M.5. *Random Generation of Local Graphs.* For computer accessing of deductive consequences from text bases involving large and/or large numbers of local graphs, it is convenient to use random selections of local graphs rather than their totalities. This expedient arises from practical rather than theoretical concerns; namely that, in general, global graphs of sentences or of sets of sentences can be so large as to exceed machine storage or processing capabilities.

M.6. *Random Generation of Local Graphs in Elementary Algebra.*
M.6.1. In the algebraic case, we adopt the idealized assumption that points on the plane, say, can be uniquely identified by some scanning device. This assumption is needed for the plane since its points are dense, but will not be needed in our own development here for the case of natural language and the graphing system we will introduce.]
M.6.2. Let us suppose that X is the set consisting of the two equations, "y = x" and "$y = x^2$", and that Y is the inequality “x ≤ 1.” It is clear that Y is deducible from X, and that this can be proven by ordinary sequential derivation techniques. Let us consider proving this graphically: Now the global graph G(X) of X is the intersection of the ordinary Cartesian graphs of "y = x" and "$y = x^2$", which is to say, the global graph G(X) consists of the two points P(<0, 0>) and P(<1, 1>). And the global graph G(not-Y) consists of all those points <a, b> such that a > 1, which is to say, the region of the plane lying to the right of the vertical line “x = 1.” By *total graphical deduction*, we would determine that

every p in G(X) and every q in G(not-Y), p and q were not identical. That is, we would determine that each of P(<0, 0>) and P(<1, 1>) was not identical with any point to the right of the vertical line "x = 1." By an instance of *partial graphical deduction*, we would do this for one of the two points comprising G(X) chosen at random, say the point P(<0, 0>) and for, say, two points P(<2, 1>) and P(<3, -2>) comprising G(not-Y) chosen at random, we would determine that these two points were not identical. This could be done simultaneously for any number of random selections of points from G(X) and G(not-Y).

M.7. *General remarks on the utility of graphs.* There are typically a myriad of “deductively relevant parts and connections” to consider in any deduction. In order to “track” these in a manner that enables near-instantaneous deductive determinations, these parts and their connections must be identified and processed essentially instantaneously. This is impossible for typical instances of syntactically represented sentences, but is very feasible for semantically represented sentences, especially when cast in the form of graphs of their denotations. Such graphs palpably illustrate the patterns formed by these “myriad connections” that relate patterns within the graphic structure of text bases to patterns within the graphic structure of their deductive consequences.

M.8. *Underlying Theory of Language.* The notions of graph of a sentence and graph of a set of sentences as used in this paper are based on a study of the semantic structure of a certain class of natural languages, referred to as "thing-relation" languages, to which English belongs. This study is detailed in the unpublished manuscript [ATR] by the author which details the way that denotations of sentences and text bases are defined in terms of the interpretations under which they are true, and the way that graphs representing these denotations are defined from them. In this paper, we primarily restrict our discussion to these representing graphs, and only sketch the semantic notions developed in ATR on which they are based.

M.9. *Generalization to a probabilistic mechanism.*We can generalize the proposed deductive mechanism to a type of probabilistic mechanism by: (1) respectively relaxing the phrases "is a subgraph of G(X)" and "is inconsistent with G(X)" to read: "is a subgraph of a weighted proportion p of G(X)" and "is inconsistent with a weighted proportion p of G(X)," and (2) replacing the phrase "deductive consequence" by "probabilistic consequence of degree p."

M.10. *The Graphing Theorem.* The graphing theorem provides a basis for the negative deductive paradigm. Under a suitable definition of the graphing function G which assigns graphs to denotations of sentences relative to interpretations, the following can be shown to hold for all languages L capable of supporting this definition:

<u>Lemma A for Graphing Theorem:</u> For all interpretations f and for all sentences or sets of sentences W, G(f/W) is the empty graph if and only if W is false under f or, equivalently, G(f/W) is a non-empty graph if and only if W is true under f.

Lemma B for Graphing Theorem: For all interpretations f, f', and for all sentences or sets of sentences W, Z, G(f/W) is graphically compatible with G(f'/Z) if and only if there is some interpretation f'' such that W is true under f'' and Z is also true under f''.

Lemma C for Graphing Theorem: For all interpretations f, f', and for all sentences or sets of sentences W, Z, G(f/W) is graphically incompatible with G(f'/Z) if and only there is no interpretation f'' such that W is true under f'' and Z is also true under f''.

The Graphing Theorem: If X is a set of sentences of L and Y is a sentence of L, then Y is a deductive consequence of X if and only if, for all interpretations f, f'' for L, G(f/not-Y) is graphically incompatible with G(f'/X).

Proof of Graphing Theorem:

Assume that the left hand side of the Graphing Theorem holds and show that the right hand side must hold as well. Accordingly, assume that Y is a deductive consequence of X; then it remains to show that for all interpretations f, f' for L, G(f/not-Y) is graphically incompatible with G(f'/X). But the assumption of the left hand side means that for all interpretations f'', if X is true under f'', then so is Y, so that for all interpretations f'' if X is true under f'' then not-Y is false under f'', i.e., that there is no interpretation f'' such that X is true under f'' and not-Y is also true under f''. But then, by Lemma, C, this is equivalent to the following: for all interpretations f, f', G(f/X) is graphically incompatible with G(f'/not-Y), which is equivalent to the right hand side of the Graphing Theorem.

Now assume that if the right hand side of the Graphing Theorem holds, then the left hand side must hold as well. Accordingly, assume that for all interpretations f, f' for L, G(f/not-Y) is graphically incompatible with G(f'/X). Then, by Lemma C, there is no interpretation f'' such that X is true under f'' and not-Y is true under f''. But that means that, for all interpretations f'', if X is true under f'', then not-Y is false under f'', that is, if X is true under f'', then Y is also true under f''. That is, Y is a deductive consequence of X.

M.12. *Special Case of the graphing theorem.* We get an interesting special case of the Graphing Theorem if we restrict ourselves to graphs that we might call "directly comparable" in the following sense: If Y and Z are sentences of L, then the graphs G(Y) and G(Z) of Y and Z respectively are *directly comparable* if and only if for all interpretations f for L, Y and Z are both true under f if and only if G(f/Y) = G(f/Z).

We note that the graphs of elementary algebra are directly comparable graphs.

M.13. *Corollary 1 of graphing theorem.* If graphs of all sentences of L are directly comparable and if X is a set of sentences of L and Y is a sentence of L, then the graph of

X is the intersection of the graphs of all sentences Z which are members of X, and Y is a deductive consequence of X if and only if G(X) is a subgraph of G(Y).

M.14. *A key equivalence revisited.* Recall that in Section 1.1. we had asserted that under suitable generalizations of the notions of "solution set" and "graph," and of the notion of "graphical distinctness" to "graphical incompatibility," that the following equivalence could be shown to hold for English sentences: *an English sentence Y is a deductive consequence of a set X of English sentences of and only if every point in the graph of the solution set of X is graphically incompatible with every point in the graph of the solution set of not-Y.*

It can be seen that the Graphing Theorem is equivalent to this key equivalence, given the following generalizations of its contained notions:

(1) A *solution of a sentence Z* is its denotation relative to an interpretation under which Z is true, and a *solution of a set X of sentences* is its denotation relative to an interpretation under which all the sentences of X are simultaneously true.

(2) A *solution set of a sentence Z or of a set X of sentences i*s the set of its solutions.

(3) The *graph of a solution of a sentence Z* is the graph G(f/Z) relative to the interpretation f under which Z is true, and the *graph of a solution of a set X of sentences* is the graph G(f/X) relative to the interpretation f under which Z is true.

(4) The *graph of the solution set of a sentence of set of sentences Z* is a connected array G(Z) of the graphs of the individual solutions of Z.

(5) If Y, Z are sentences or sets of sentences of L, and if f and f' are interpretations for L, then the graphs G(f/Y), G(f'/Z) are graphically incompatible if and only if the graphing conventions for the graphing system are such that there is no interpretation f'' such that both Y and Z are true under f''.

M.15. *Graphing systems*.
15.1. *Graphing systems: a definition*.
A *graphing system G* for a language L relative to a set of permissible interpretations for sentences of L which assign denotations to sentences and to sets of sentences of L as well as truth conditions which specify when a sentence is true, and relative to a graphing function G which assigns to every sentence and set of sentences Z of L relative to every interpretation under which it is true a visual icon which is, in principle, inter-retrievable with the denotation of that sentence, and from either of which it can be verified that the truth condition of that sentence relative to that interpretation is satisfied.
15.2. *Directly comparable graphing systems: a definition.* A graphing system G is a *directly comparable graphing system for L* if any two sentences of L are directly comparable. (See M.12 above for the definition of direct comparability)
15.3. *Extensibility of graphing systems.*

(i) If G is a graphing system for L, then G can be extended to a directly comparable graphing system G^ for L such that G* is obtained from G by cylindrification. (ii) If G^ is a directly comparable graphing system for L, then G* can be collapsed into a non-directly comparable graphing system for L such that G is obtained from G^ by projection.
15.4. *Cylindrification G^ of the Graphing System G.*
(i) If f is an interpretation for L and Y is a sentence, the cylindrification of Y relative to L, in symbols CYL(f/Y), is defined as the empty set if Y is not true under f and is defined as the set: {f(Z)/ Z is true under f}, otherwise, that is, if Y is true under f.
(ii) If f is an interpretation for L and X is a premise set, the cylindrification of X relative to L, in symbols CYL(f/X), is defined as the empty set if some sentence Z in X fails to be true under f, and is defined as the set: {f(Z)/ Z is true under f}, otherwise, that is if all sentences in X are true under f.
(iii) The graph G^(CYL(f/Y) of CYL(f/Y) is defined as the connected array of the Graphs G(f/Z), where f is such that Y is true under f, and Z ++ranges over all sentences of L which are true under f.
(iv) The graph G^(CYL(f/X) of CYL(f/X) is defined as the connected array of the Graphs G(f/Z), where f is such that all sentences of X are true under f and Z ranges over all sentences of L which are true under f.
The cylindrification G^ of G is a directly comparable system for L.
15.5. *Consequence of these definitions and corollary 1.*
then Y is a deductive consequence of X if and only if G^(CYL(X)) is a subgraph of G^(CYL(Y)).
15.6. *Graphing systems for simple vs complex languages.*
For simple languages like the languages of elementary algebra and traditional categorical logic, directly comparable graphing systems provide the best basis for the implementation of the Graphing Theorem for graph-based machine deduction. For complex languages such as natural language, non-directly comparable graphing systems provide the best basis of the implementation of the Graphing Theorem for complete graph-based machine deduction.

M.16. The usual rectangular Cartesian graphs of elementary algebra are directly comparable in the indicated sense inasmuch as: two equations or inequalities in the same 2 variables have a common pair of real numbers as their "solution" (in our terminology, they have a common "denotation under some interpretation") if and only if they are graphed as the same point on the plane. Then, by Corollary 1 of the Graphing Theorem, we have the following: A given equation or inequality Y restricted to two variables is a deductive consequence of a given set X of equations and inequalities restricted to those same two variables if and only if the usual rectangular Cartesian global graph of X is a subgraph of the usual rectangular Cartesian global graph of Y, i.e., if and only if every point on the global graph of X is already a point on the global graph of Y. Note that this result generalizes directly to other dimensions, but is not true for polar global graphs for any dimension since equations or inequalities can have different denotations and yet be graphed into the same point.

Appendix N. Notes on Some Related Issues

N.1. *Herbrand/Skolem Techniques: Brute Force Deduction.*
There are various techniques known from the logical literature whereby a set K of sentences of an underlying predicate logic and a possible consequence S of K are respectively re-cast, first in open quantifier-free forms S* and K* and then into sets S*' and K*' each consisting of closed quantifier-free formulas - i.e., sentences - obtained by substituting terms for variables in S* and K* in all possible ways. If all validating truth functions on K*' are also validating truth functions on S*', then S is considered to be a deductive consequence of K. If the quantifier-free sentences belonging to K*', and S*' are formulated in a canonical disjunctive normal form, then all validating truth functions explicitly appear as disjuncts. One can imagine a string running through all possible sequences of mutually compatible disjuncts of the quantifier free sentences of K*' and attempting to see whether every such string contained a disjunct of each sentence in S*'. One can further imagine all possible such strings being run at the same time, which is possible since different strings would be independent. This means, then, that it would be fully possible - for a deity, say - to run all these strings at the same time and across the entirety of K*'. The obvious difficulty with such a procedure is that there generally are *infinitely* many formulas in the sets K*' and S*', hence infinitely many strings of the kind described. Thus, in spite of the potentiality of massively parallel execution, this sort of approach would be beyond machine computation.

N. 2. *Herbrand/Skolem Techniques: Resolution Logic Deduction*
The currently used techniques [1] dating from those of Robinson [12] use an alternative procedure based on Herbrand/Skolem techniques which is machine executable, but which is no longer parallel. The approach taken is to attempt to prove that S is a deductive consequence of K by deriving - through the use of special "resolution-based" inference rules applied to the combined sets K* and [not-S]* - to derive some contradictory subset of the combined sets K*' and [not-S]*', a circumstance that would prove that S was a deductive consequence of K. We note how this way of using Herbrand/Skolem techniques differs from the "brute force" method described above: first, it operates on the finite set made up of K* and S*, rather than directly on the infinite set K*'; second, it deals with formulas directly rather than with their truth functions; third, it suffers from the usual deficiencies of inference-rule-based systems in that the rules are interdependent and the order in which they are to be applied in any particular case is indeterminate. Its operations are not therefore executable in a massively parallel manner, but instead require a variety of heuristic strategies to sort out the "best" order of rule application.

N.3. *Generalizing an Apparent Human Deductive Capability*. Humans appear able to recognize deductive connections among simple NL sentences near instantaneously, a circumstance which suggests that humans might exercise (not necessarily consciously) a deductive mechanism for doing this, but one which does not extend to more complex NL sentences, perhaps owing to psychological limitations. The model we propose attempts to describe a mechanism which might not only account for this apparent human capability and limitations, but which also generalizes to more complex NL sentences for near instantaneous machine execution without such limitations.

N.4. *Generalizing on an Apparent Human Probabilistic Capability.* Humans also appear capable of near-instantaneously making reasonably accurate probabilistic estimates in simple situations. An example of the latter would be if there were ten men and 100 mortals, the probability that Socrates was a man based on the joint circumstance that Socrates was mortal and that all men were mortal, would be one-tenth, i.e., the proportion of men who were mortals (i.e., Socrates might be an alligator). And the probability that Socrates was not a man - made on the same basis - would be nine tenths, i.e., the proportion of mortals that were not men. As remarked in Section M.9., above, our model can generalize this capability to handle more complex situations by generalizing the proposed deductive mechanism to a type of probabilistic mechanism by relaxing the phrases "is a subgraph of G(X)" and "is inconsistent with G(X)" to read: "is a subgraph of a weighted proportion p of G(X)" and "is inconsistent with a weighted proportion p of G(X)," and (2) replacing the phrase "deductive consequence" by "probabilistic consequence of degree p."

N. 5. *Constraints in Serial Deduction.* Serial approaches to machine deduction are constrained: (1) to execute the required multiple subtasks involved in a given deduction in a *serial* rather than parallel manner (i.e., completing one subtask before initiating another), (2) to execute these subtasks in an order which is not fully determinate, and (3) to execute those subtasks on those selected parts of memory that are pre-determined to be most relevant to sought-for a deductive consequence. Serial execution of required subtasks (constraint (1))would not necessarily slow down deduction; but the indeterminacy posed by constraints (2) and (3) would inasmuch as they require auxiliary trial-and-error "heuristic" techniques to determine the "right" place to start the inference mechanism and the "right" sequence of operations to carry it through. It is these latter determinations that slow the inference process down to the point where it is impractical to use machines for general deductive purposes.

N. 6. *Summary of Proposed Deductive and Probabilistic Models.* For every sentence S and for every interpretation f under which S is true, we define the global graph G(f/S) as the denotation which f assigns to S. (2) For every text base T and for every interpretation f of T under which all sentences in the text base are true, we define the global graph G(f/T) as the denotation which f assigns to T, and which is constructed as a linked array of all the consistent local graphs G(f/S) in G(f/T) as S ranges over sentences of T. (3) We then define the global graph G(S) of a sentence S as a linked array of the local graphs of G(f/S), as f ranges over all interpretations of S under which S is true, and similarly define the global graph G(T) of the text base T as a linked array of all local graphs in the global graph G(f/T), as f ranges over all interpretations under which all the sentences in T are true. (4) Finally, a sentence S is defined as a deductive consequence (probabilistic consequence of degree r) of a text base T if either: (i) for all (a weighted proportion r of) the subgraphs G(f/T) of G(T), some subgraph G(f/S) of G(S) has an embedded image within G(f/T), or (ii) for all (a weighted proportion r of) the subgraphs G(f/T) of G(T), some subgraph G(f/not-S) of G(not-S) is inconsistent with G(T); either circumstance is determinable by a machine.

N. 7. *Randomizing global graphs*. Because global graphs typically get very large, it is often necessary to restrict the number of local graphs which they contain, a restriction which results in loss of accuracy in certain cases. In order to minimize such a loss in accuracy, we randomize the attempted matches, rather than limit the number of local graphs considered. That is, instead of matching *all* local graphs in G(S) (or in G(not-S)) against *all* local graphs in G(T), a machine matches randomly generated local graphs among them.

N. 8. *Focus of Proposed Model is on Machine Deduction, not Human Deduction.* While human ability to near instantaneously execute some simple deductions provided the motivation to pursue our line of research leading to the model for machine deduction presented here, our primary focus has been on machine rather than human deduction, There is a large body of research in human deduction directed to accounting for various human capabilities in executing certain deductive operations. This research is essentially empirical, hence more on the psychology of human reasoning than on machine reasoning. Two areas of research can be distinguished as “mental model” and “mental logic” theories of human reasoning, each encompassing both deductive and a certain type of probabilistic reasoning in humans.

N. 9. *Comparison between the Proposed Model and "Mental Model" Theories of Human Reasoning.* There are some points of contact between the proposed model and mental model theories of human reasoning as they apply to deduction, developed by David Johnson-Laird and others. [8], [9], [10]. We comment briefly on what is similar in the two theories and what is different. Mental models as defined by Johnson-Laird are variously understood as models of how a person "mentally represents and operates on" logical relationships among entities and relations expressed in given sentences to determine when certain of them are deductive consequences of – or incompatible with – others, hence can operate in both a “positive” and “negative” mode. Like our proposed model, they offer a semantic (rather than proof-theoretic) model of reasoning. There are important differences, however: (i) our model is designed to automate computer reasoning, whereas mental models are designed to account for certain types of human reasoning; (ii) our model uses the same mechanism for all cases, and depends only on the logical structure of the sentences being processed whereas mental models appear to depend on the kinds of sentences being processed; and (iii) the mechanism used in our proposed model applies to all sentences, whereas the mechanism used in “mental models” varies as sentences vary.

N. 10. *Comparison between the Proposed Model and “Mental Logic” Theories of Human Reasoning.* Mental logic theories, like mental model theories and unlike our proposed model, are designed to explain human rather than machine reasoning. [3]. But, unlike both our proposed model and mental model theories, mental logic approaches are proof theoretic rather than semantic in character. Mental logic approaches, like those of our proposed model and like the approach of mental models, have both a positive and negative mode of operation, but in a proof theoretic format. In a positive deductive mode, mental logic mechanisms sequentially apply selected deductive operations to a syntactic representation X' of given set X of sentences (premises), the goal of which is to reach the syntactic representation Y' of a given sentence Y as end result. If Y' is reached in this manner, Y is regarded to be a deductive consequence of X. In a negative deductive mode, mental logic mechanisms sequentially apply selected deductive operations to a syntactic representation X' of X together with a syntactic representation (not-Y)’ of the negation (not-Y) of Y, X’ and (not-Y)’ collectively taken as premises, the goal of which is to reach a contradiction as end result. If a contradiction is reached in this manner, Y is regarded to be a deductive consequence of X. Thus in either mode, the end result is that Y is determined to be a deductive consequence of X. While evidence from experiments on human reasoning tend to support the mental model type of explanation of human reasoning over the mental logic type of explanation of that reasoning, the mechanics of mental logic explanations of reasoning are generally clearer than the mechanics of mental model explanations. For, while the rationale underlying different examples of mental model based reasoning is intuitively clear in examples, there is no precise general account of mental model reasoning which would be comparable in precision or generality to accounts of mental logic reasoning. That is, while it is fairly clear what internalized forms of proof theoretic based reasoning would be like, it is not as clear what internalized forms of semantically based reasoning would be like.

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